Optimal Delegation of Sequential Decisions: The Role of Communication and Reputation*

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Abstract

We analyze the delegation of a set of decisions over time by an informed principal to a potentially biased agent. Each period the principal observes a state of the world and sends a "cheap-talk" message to the agent, who is privately informed about her bias. We focus on principal-optimal equilibria that satisfy a Markovian property and show that if the potential bias is large, then the principal assigns less important decisions in the beginning and increases the importance of decisions towards the end. In the beginning of their relationship, the biased agent acts exactly in accordance with the principal's preferences, while towards the end, she starts playing her own favorite action with positive probability and gradually builds up her reputation. Principal provides full information in every period as long as he has always observed his favorite actions in the past. If we interpret the evolution of the importance of decisions over time as the career path of an agent, this finding fits the casual observation that an agent's career usually progresses by making more and more important decisions and provides a novel explanation for why this is optimal. We also show that the bigger the potential conflict of interest, the lower the initial rank and the faster the promotion.

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1. INTRODUCTION

Consider a principal, say a career bureaucrat, who needs to delegate a series of operational decisions to an agent, say a newly hired subordinate. The principal is more informed about the policy issues involved and has an opportunity to communicate these issues to the agent before she makes a decision. However, the agent could be biased and whether she is biased or not is her private information. How should the principal sequence these decisions? More important ones first or less important ones? Similarly, we could think of an informed investment advisor who gives advice to an investor, who might have some behavioral bias. How should the advisor present investment opportunities? More important ones first or less? A big bunch of them at the beginning or only a few?

We could also think of the problem faced by the principal as the optimal design of an agent's career path. At what level of the hierarchy should the principal start the agent and how should he go about promoting her? Is it best to start her at a very low rank and keep her there for a long time, or should the career of the agent progress at a steady pace? What is the role of potential conflict of interest between the principal and the agent in the optimal design of the career path of the agent?

In our model, there is a principal who needs to delegate a set of decisions to an agent over finitely many periods and some of these decisions might be more important than the others. Each period the principal decides which decision (or set of decisions) to delegate to the agent in that period. He then observes the relevant state of the world for that period and communicates this information to the agent. The agent observes the message sent by the principal and makes a decision and the decision is revealed. State of the world and the decision jointly determine the payoffs in each period. Overall payoff of each player is equal to the weighted sum of period payoffs, where the weight of each period is determined by the importance of the decisions made in that period. The principal would like the decision to match the state of the world while the agent might be biased. More crucially, the principal's preferences are common knowledge while that of the agent is her private information.

We assume that the information on the state of the world is "soft," i.e., it cannot be verified, and that the messages are costless. This makes the communication phase in each period a "cheap-talk" game, i.e., the principal may lie and this has no direct costs for him. We also assume that the decisions of the agent are not contractible. This could be due to legal reasons, as in the example of a bureaucrat and a subordinate, or because the decisions are impossible to reproduce before courts.[1] Our third crucial assumption is that states of the world are independently distributed across periods. This implies that the principal decides how much information to reveal each period without having to worry about its informational implications for the future states. Finally, we assume that the agent's preferences are similar for each decision, i.e., she either shares the preferences of the principals or is biased in the same manner for all the decisions. Therefore, our model is more suitable for situations in which the decisions are related, such as a series of investment decisions, or budgetary decisions for different departments, etc.

We assume that the agent is either an unbiased type, who myopically chooses the decision best suited to the state given her beliefs in each period, or a biased type who acts strategically. The un-

[1] For example, the decision might be how much time to allocate to a certain task, or how much to invest in human capital, which might be observable by the principal, but still impossible to verify. The assumption that decisions are observable but not contractible follows the "incomplete contracts" perspective (e.g., Grossman and Hart (1986) and Hart and Moore (1990) and is a standard one in the "optimal delegation" literature that we discuss in Section 2 (see, for example, Holmström (1977) and Dessein (2002)).

biased type resembles a commitment type that is common in reputation literature. But, unlike a standard commitment type who always plays the same action, the unbiased agent plays a best response to her beliefs in any given period. Our aim is to characterize the perfect Bayesian equilibria of the resulting extensive form game with incomplete information.

In order to gain some intuition about the major forces at work in the model, note that the principal would like to receive his favorite decision, i.e., the unbiased decision which matches the state, in each period. Therefore, if he believes that the agent is going to make the unbiased decision with high enough probability, then he has an incentive to reveal the state of the world truthfully. The biased agent, on the other hand, would like to make a decision that is best for her, i.e., the biased decision, in any period and for that reason she would like to receive accurate information. However, if she makes a decision that is different from the decision that would be made by the unbiased commitment type, she would be revealed as biased and receive no information in the future. This introduces reputation concerns in the sense that she may masquerade as the unbiased agent and act against her own interest today, in order to receive better information in the future.

It is clear that the agent benefits from truthful communication.It turns out that, ex-ante, the principal benefits from truthful communication as well, irrespective of whether the agent is biased or not. In fact, if he could commit to a communication strategy before learning the state, he would commit to full revelation. Therefore, the stage game exhibits both conflict of interest, because of the possible bias, and common interest, because of the common preference for truthful communication.

These considerations imply that the principal may choose the allocation of decisions in a strategic manner in order to exploit the reputational concerns of the agent and facilitate communication. In particular, if he assigns relatively more important decisions to the future, then the biased agent may choose to play the unbiased action early on in the game, which benefits the principal both because he prefers the unbiased action and it enables truthful communication. At the same time, he would not like to leave too many important decisions to later periods because the biased agent will surely play the biased action at the end. This creates a trade-off in his choice of the allocation of decisions. We would like to understand how this trade-off is resolved in equilibrium.

As is usual in cheap-talk games, our model exhibits multiple equilibria. In order to circumvent this problem, we focus on equilibria that satisfy a Markovian property and yield the highest expected payoff for the principal, which we call the *principal-optimal equilibria*. We show that, in any principal-optimal equilibrium, if the potential bias is large enough and the initial reputation of the agent is not very good, then the principal assigns less important decisions in the beginning and increases the importance of the decisions towards the end. If there are sufficiently many periods, then in the beginning of their relationship, the biased agent acts exactly in accordance with the principal's preferences, while towards the end, she starts mixing, i.e., playing her own favorite action with positive probability, and gradually builds up her reputation. Interestingly, once the agent starts mixing, she builds reputation at just the right speed in order to facilitate communication in every period: If her reputation were to evolve faster, then truthful communication would fail in the current period, while if it were to evolve slower, it would fail in the future. Principal reveals the state truthfully in every period as long as he has always observed the unbiased action in the past. If, in contrast, the potential bias of the agent is small, then the principal chooses to assign more important decisions early on in their relationship.

The main reason behind this result is roughly as follows. Since the biased agent will definitely play the biased action at the end of the game, the principal would like to leave as little as possible to the later periods of the game. However, doing so distorts the reputational incentives of the agent and causes her to play the biased action early on. If the potential bias is large, then future must be important enough in order to provide reputational incentives, which leads to increasing importance over time, while if it is small, then these incentives can be provided even if the future is not very important.

We also show that, for sufficiently bad initial reputation levels, the principal-optimal equilibrium is also agent-optimal, i.e., the equilibrium that we focus on Pareto dominates all other equilibria. Furthermore, since the principal fully reveals the state in every period as long as he observes the unbiased action, a principal-optimal equilibrium is also the most informative one on the equilibrium path.[2] We therefore believe that principal-optimality is a reasonable equilibrium selection criterion.

Our results imply that as the potential conflict of interest between the principal and the agent increases, initial decisions become less important but their growth rate increases, i.e., if the potential bias is large, then promotion takes place faster and if it is small, importance of the decisions decreases at a slower rate. Finally, we show that, if there is a large number of decisions and the principal can choose the number of periods over which to allocate these decisions, she would prefer as many periods as possible. In other words, if the potential bias is large, then the principal would prefer to give the agent trivial tasks for a long period of time and then promote her quickly towards the end of her career.

We believe that our main findings are in line with causal observations. Usually an agent starts her career in an organization by making less important decisions and is gradually promoted to make more and more important decisions. Of course, there could be many reasons why this is the case, including on the job training, testing the skills of the agent, etc. In this paper, we provide another rationale, which is based on disciplining a possibly biased agent to act in the interest of the principal and maintaining a healthy communication between them. Also, causal empirics suggest that a newly hired agent with some history of past decisions (e.g., in another institution) would presumably have a lower potential conflict of interest (for otherwise he would not be hired) and accordingly start at a higher rank than an agent with no history. Still, the latter might be promoted at a faster rate as long as her decisions turn out to be in the interest of the principal.

2. Related Literature

The main question analyzed in this paper, i.e., optimal sequential delegation, seems to be novel, but "gradualism" (or "starting small") has been a recurrent theme in economics. The most relevant papers in that regard are Watson (1999, 2002) and Andreoni and Samuelson (2006).[3] Watson (1999) and Watson (2002) analyze an infinitely repeated prisoners' dilemma type game with incomplete information and variable stakes over time. In the stage game, the "low" type player prefers to "betray," which benefits herself, injures the other player, and ends the game, while the "high" type prefers cooperation as long as the other player also cooperates. He shows that starting with small stakes supports perpetual cooperation between the high types as an equilibrium outcome. Watson (2002) assumes

[2] As we will show in the sequel, this is true except perhaps in the first period. We refer the reader to Chen et al. (2008) and the references cited therein for justification of the most informativeness criterion in cheap-talk games.

[3] Other papers that feature gradualism as an optimal or equilibrium outcome include Marx and Matthews (2000), Blonski and Probst (2004), and the loan model in Section 6 of Sobel (1985).

that players commit to the way stakes change over time while Watson (1999) characterizes an equilibrium in which the stakes satisfy a renegotiation condition. Andreoni and Samuelson (2006) study a twice repeated prisoners' dilemma game with incomplete information and variable stakes. Players are conditional cooperators in the sense that, in the stage game, type-α player prefers to cooperate if she believes that the other player cooperates with at least α probability.[4] They characterize the equilibria of this game with exogenously given stakes and show that "starting small" leads to the best payoffs for the players.[5] The main point of departure of our model from these papers is that, in contrast to a prisoners' dilemma game, our stage game is a game of strategic communication that exhibits common interest as well as conflict of interest. Furthermore, we assume that one of the players has the authority to determine the stakes involved in their relationship and analyze how they are determined in equilibrium. Finally, we show that gradualism is not always the optimal arrangement for the principal. Indeed, if the potential conflict of interest between the agent and the principal is small enough, then the opposite arrangement of "starting big" turns out to be optimal for the principal.

The question of optimal delegation of decisions has been first studied by Holmström (1977). He analyzes a model in which a principal who is unable to commit to outcome contingent transfers faces an informed but biased agent. In equilibrium, the principal chooses a set of actions and gives the agent the authority to choose an action from this set. Optimal delegation reflects the trade off between the need to give flexibility to the agent in order to take advantage of her superior information and the need to restrict her freedom in order to avoid her opportunism.[6] Our model differs from the models in this literature in three important aspects: (1) It is the principal who is informed about the state of the world; (2) The agent's bias is her private information; (3) Delegation problem is dynamic and concerns the optimal sequencing of decisions with respect to their importance rather than a static one that concerns how much flexibility to give to the agent.

In each period of our model, the principal and the agent are involved in a cheap-talk game, which has been introduced by Crawford and Sobel (1982). They analyze the equilibrium communication behavior between an informed but biased sender and an uninformed receiver and show that the informativeness of equilibrium decreases in the degree of the sender's bias. There are two main differences between Crawford and Sobel (1982) and our model: (1) The degree of preference divergence between the sender and receiver is the private information of the receiver; (2) The game is repeated, where in each period a new state of the world is realized but preferences remain the same.

Morris (2001) also differs from Crawford and Sobel along those two dimensions. The main difference is that in Morris (2001) the bias is the private information of the sender whereas in our model it is the private information of the receiver. Morris (2001) finds that the unbiased sender, who prefers to inform the receiver about the state of the world, may choose not to do so in the first period in order to be regarded as unbiased and hence better inform the receiver in the future. In contrast, in our model, the biased receiver my mimic the unbiased receiver in order to maintain a good reputation and receive better information in the future. Furthermore, we analyze the optimal sequencing of decisions

[4] They allow α to be negative or greater than one, which corresponds to unconditional cooperators or defectors, respectively.

[5] Andreoni and Samuelson (2006) also test their theory experimentally and find empirical support for their predictions. Andreoni et al. (2016) extend this paper so that players choose the stakes themselves in the experiment. They show that the subjects indeed choose the payoff maximizing strategy of starting small.

[6] Holmström's findings have further been generalized by Alonso and Matouschek (2008) and Amador and Bagwell (2013a,b).

by the sender (i.e., the principal).

Morgan and Stocken (2003) analyzes a one period cheap-talk game with a sender with uncertain preferences, whereas Sobel (1985) and Benabou and Laroque (1992) are earlier papers that analyze repeated cheap-talk games, except that they assume that the unbiased (or good) sender always tells the truth. Li and Madarász (2008) extend Morgan and Stocken (2003) so that the bias can be in either direction and compare equilibria under known and unknown biases, while Dimitrakas and Sarafidis (2005) allow the bias to have an arbitrary distribution. Our model differs from these papers in that we assume the bias is receiver's private information and that the cheap-talk game is repeated.

Another related paper is Ottaviani and Sørensen (2001) in which a sequence of privately informed experts, who are exclusively concerned about their reputation for being well-informed, offer public advice to an uninformed agent. They show that reputational concerns may lead to herding by the experts.[7] Our model can also be framed as a model of sequential cheap-talk with multiple experts (principals) but we have an agent who is privately informed about the preference divergence between herself and the experts, and it is the agent who is concerned about reputation.[8]

Optimal delegation rules have also been studied by Dessein (2002) within a one-shot cheap-talk game, in which an uninformed principal decides whether to delegate the decision making authority to an informed but biased agent. He shows that decentralization is better as long as the bias is not too large relative to the decision maker's uncertainty about the state of the world.[9] In our model, the principal is informed and the agent's preferences are private information, Furthermore, there are multiple rounds of cheap-talk games and the delegation question pertains to the optimal sequencing of decisions over time.

Our work is also related to the literature on pandering. Maskin and Tirole (2004) analyze a two-period model where in the first period an official chooses a policy, which determines whether she stays in office in the second period. They show that if the official's desire to stay in office is sufficiently strong, then in the first period she could choose a popular action, i.e., she could pander to public opinion even if she does not think that the public opinion is the optimal policy. In our model, incentives to pander come from the desire to receive better information rather than the desire to stay in office.[10]

Another related strand of literature is the one on career concerns pioneered by Holmström (1999), in which an employee's concern about her reputation for talent leads her to exert costly effort even without explicit incentives provided by a contract.[11] In our model, concern for reputation for being unbiased arises from the agent's incentives to obtain accurate information and leads her to act in the interest of the principal.

[7] Also see Ottaviani and Sørensen (2006a,b) in which an expert with reputational concerns (but no bias) fails to provide full information to the receiver.

[8] There are other models in which multiple experts with known biases are involved in simultaneous or sequential cheap-talk, among which are Gilligan and Krehbiel (1989), Austen-Smith (1990), and Krishna and Morgan (2001).

[9] Alonso et al. (2008) and Rantakari (2008) analyze the same question when there are more than one privately informed and biased agent.

[10] Brandenburger and Polak (1996), Vidal and Möller (2007), Acemoglu et al. (2013), Che et al. (2013), and Morelli and Van Weelden (2013) are some of the other papers in the pandering literature.

[11] Holmstrom's model was originally developed in a paper published in 1982 in an edited book. See also Holmström and Costa (1986).

3. The Model

A principal needs to delegate a decision to a potentially biased agent in each of N periods. In what follows, it is more convenient to count the periods in reverse, so that the period in which the first decision is made is labeled N, the second $N-1$, and so on. The agent's type $\beta \in \{0, b\}$, where $b > 0$, is her private information and she is biased, i.e., $\beta = b$, with probability $p \in (0,1)$. With the remaining probability, the agent is an unbiased commitment type, i.e., $\beta = 0$. We provide more details about these types further below. In each period i of the N period game, the following stage-game is played:

1. The principal chooses the parameter $\delta_i \in [0,1]$ for period i. The parameter δ_i represents the proportion of the remaining decisions deferred to subsequent periods and $1-\delta_i$ represents the proportion made in period i. Since there are no subsequent periods in the last period, we set $\delta_1 = 0$.

2. Nature chooses the state of the world $\theta_i \in \{0,1\}$. We assume that each state is equally likely and that states are independent across periods.

3. The principal observes the state of nature θ_i and chooses a message $m_i \in \{0,1\}$.

4. The agent observes the principal's message and chooses an action $a_i \in \mathbb{R}$ without observing the state of the world.

We define the importance parameter γ_i for period i as the proportion of decisions made in period i. More precisely, $\gamma_N = 1-\delta_N$, $\gamma_i = \delta_N \cdots \delta_{i+1}(1-\delta_i)$, and $\gamma_1 = \delta_N \cdots \delta_2(1-\delta_1)$. If the importance parameter for the period is γ_i and the agent plays $a_i \in \mathbb{R}$, then the principal's payoff for the period is $v(a_i, \theta_i, \gamma_i) = -\gamma_i(a_i - \theta_i)^2$ while the biased agent's payoff for the period is $u(a_i, \theta_i, b, \gamma_i) = -\gamma_i(a_i - (\theta_i + b))^2$. The parameter $b > 0$, measures the divergence of the preferences of the principal and the agent, or simply the "bias" of the agent. The payoff of each player over the N periods is simply the sum of the payoffs from each period.

The state of the world, the messages, and the decisions of the agent are unverifiable and hence cannot be contracted upon. Furthermore, as the payoff functions imply, the messages have no direct payoff consequence. This implies that the communication between the principal and the agent is "cheap-talk" and that outcome contingent contacts cannot be written. After the period is over, the principal and agent observe their payoffs and therefore the agent learns the state in period i and the principal learns the agent's action.

We assume that the unbiased agent is a commitment type who, in each period, plays the action that is perfectly aligned with the principal's preferences. More precisely, fix a period i and let $\lambda \in [0,1]$ be the probability assigned by the agent to the event that $\theta_i = 1$. Define the *best period action for type* $\beta \in \{0, b\}$ as follows:

$$a^\beta(\lambda) = \underset{a_i}{\operatorname{argmax}} \mathrm{E}\left[-(a_i - (\theta_i + \beta))^2\right] = \lambda + \beta.$$

We refer to $a^0(\lambda)$ as the *unbiased action* and to $a^b(\lambda)$ as the *biased action.* The unbiased agent is a commitment type (or an automaton) who plays action $a^0(\lambda)$ in each period, i.e., she picks the myopic best response of an agent who has zero bias and therefore chooses an action that is perfectly aligned with the principal's preferences. The biased agent, in contrast, is rational and chooses her period action strategically.

The main question analyzed in the paper is the optimal sequencing of decisions by the principal. For a fixed set of N decisions and arbitrary importance parameters, this is a difficult problem. We analytically simplify the problem by assuming that the principal can choose any $\gamma_i \in [0,1]$ at the beginning of each period i and that at least one γ_i is positive. In other words, the principal can fine tune the importance of period i decision in any way that he likes. This could be motivated in two different ways: (1) There is a large set of decisions and each period the principal chooses which subset of these decisions to delegate; (2) There is a large set of decisions with varying importance and each period the principal chooses one decision from this set.

Let $o_i = (\delta_i, \theta_i, m_i, a_i)$ denote a period i outcome and O_i the set of all possible period i outcomes. For any $i < N$, let H_i be the set of all histories before decision i is made, i.e., sequences of the type $(o_N, \ldots, o_{i+1})$. Define $H_N = \{\emptyset\}$. The principal's belief in period i is a mapping $p_i : H_i \to [0,1]$ where $p_i(h) = \text{prob}(\beta = b|h)$ for each $h \in H_i$. A period i strategy for the principal is comprised of two components: a weight choice strategy $\tau_i : H_i \to [0,1]$ where $\tau_i(h)$ is the principal's choice of $\delta_i \in [0,1]$ in period i after history $h \in H_i$ and a communication strategy $\mu_i : H_i \times [0,1] \times \{0,1\} \to [0,1]$ where $\mu_i(h, \delta_i, \theta_i)$, denotes the probability of sending message 1 after history h, weight δ_i, and state θ_i. The agent moves after histories of the type $(h, \delta_i, \theta_i, m_i)$ where $h \in H_i$. For any history $h \in H_i$, a period i information set for the agent is given by $I_i = \{(h, \delta_i, \theta_i, m_i) : \theta_i \in \{0,1\}\}$. In other words, before making a decision in period i, the only thing that is not known by the agent is θ_i. Let the set of all period i information sets be $\mathscr{I}_i$. Agent's belief that $\theta_i = 1$ is given by $\lambda_i : \mathscr{I}_i \to [0,1]$. Since the unbiased agent is a commitment type, we will only describe strategies for the biased agent. Biased agent's (mixed) strategy is given by $\alpha_i : \mathscr{I}_i \to \Delta(\mathbb{R})$, where $\Delta(\mathbb{R})$ denotes the set of all probability distributions with support in $\mathbb{R}$. For ease of exposition we will sometimes write $\lambda_i(h, \delta_i, m_i)$ and $\alpha_i(h, \delta_i, m_i)$ for any $h \in H_i$, $\delta_i \in [0,1]$, and $m_i \in \{0,1\}$. A collection $\sigma = (\tau_i, \mu_i, \alpha_i, p_i, \lambda_i)_{i=1}^N$ constitutes an *assessment* and we focus our attention on *perfect Bayesian equilibria (PBE)* of the game that satisfy the following properties:

Property 1. Fix an assessment σ, a period i, a history $h \in H_i$, and an outcome $o_i = (\delta_i, \theta_i, m_i, a_i)$. If $a_i \neq a^0(\lambda_i(h, \delta_i, m_i))$, then σ is a PBE only if $p_j(\hat{h}) = 1$ in any period $j = i-1, \ldots, 1$ and history $\hat{h} \in H_j$ that follows o_i.

This property implies that equilibrium beliefs put full probability on the agent being the biased type (i.e., the strategic player) after histories that contain an action which is different from the unbiased action. It is automatically satisfied in a sequential equilibrium because the unbiased action is the unique action that is available to the unbiased type. However, we work with perfect Bayesian equilibria because there are certain difficulties in defining sequential equilibria for games with infinite action sets such as the game we consider in this paper.[12]

Property 2. Strategies do not depend on the past communication behavior of the principal. In other words, for any $i = N, \ldots, 1$ and histories $h, h' \in H_i$ in which $\delta_j = \delta'_j$ and $a_j = a'_j$ for $j = N, \ldots, i+1$, $\tau_i(h) = \tau_i(h')$, $\mu_i(h, \delta_i, \theta_i) = \mu_i(h', \delta_i, \theta_i)$, and $\alpha_i(h, \delta_i, m_i) = \alpha_i\left(h', \delta_i, m_i\right)$.

[12] See Myerson and Reny (2015) for a definition of sequential equilibrium in infinite action games and a discussion of the difficulties in extending the definition in a meaningful way from finite games to infinite games.

Note that the principal's past communication behavior does not affect current and future payoffs or the states of the world. Therefore, this is a Markovian property in the sense that strategies are independent of payoff irrelevant histories. In particular, this restriction eliminates punishments in the form of "no information revelation" or "playing the biased action" after histories in which the principal has lied.[13] In addition to its Markovian nature, this property is also implied by "renegotiation-proofness", because even after histories in which the principal has lied, both parties have an incentive to choose a continuation equilibrium in which there is full communication. We will comment on how our results change when this restriction is removed in Section XXX.

4. Preliminaries

As is usual in cheap-talk games, there are many equilibria of the game defined above even under the Markovian restriction introduced in Property 2. In this paper, we focus on the *principal-optimal equilibria*, i.e., equilibria that maximize the principal's expected payoff. For expositional reasons, we will also restrict attention to equilibria in which for any $i \in \{N, N-1, \ldots, 1\}$, $h \in H_i$, and $\delta \in [0,1]$: (1) The principal sends message $m = 0$ after observing $\theta = 0$, i.e., $\mu_i(h,\delta,0) = 0$; (2) The agent puts positive probability only on the biased and unbiased actions, i.e., $\alpha_i(h,\delta,m) \in \Delta\left(\left\{a^0(\lambda_i(h,\delta,m)), a^b(\lambda_i(h,\delta,m))\right\}\right)$; (3) The agent plays the biased action with the same probability after either message, i.e., the probability of $a^b(\lambda_i(h,\delta,0))$ under $\alpha_i(h,\delta,0)$ is equal to the probability of $a^b(\lambda_i(h,\delta,1))$ under $\alpha_i(h,\delta,1)$. Given these restrictions, we simplify notation and describe period i strategies by functions $\tau_i : H_i \to [0,1]$, $\mu_i : H_i \times [0,1] \to [0,1]$, and $q_i : H_i \times [0,1] \to [0,1]$, where $\tau_i(h)$ determines the principal's choice of δ, $\mu_i(h,\delta)$ determines the probability that the principal sends message 1 after choosing δ and observing $\theta_i = 1$, and the function $q_i(h,\delta)$ is a distributional strategy (see Milgrom and Weber, 1985) for the agent that determines the total probability with which the agent plays the biased action. Note that restriction (2) is satisfied in any equilibrium and restriction (1) is without loss of generality in terms of equilibrium outcomes. The proof of our main theorem establishes that the strategies in any principal-optimal equilibrium satisfy (3).

Fix an assessment σ that satisfies these three restrictions, a period i, a history $h \in H_i$, and $\delta_i \in [0,1]$. Let $\Pr(m)$ be the total probability that the principal sends message $m \in \{0,1\}$ in this assessment in period i after (h,δ_i).[14] Let $q = q_i(h,\delta_i)$ and $\lambda(m) = \lambda_i(h,\delta_i,m)$. We can then write the principal's and the agent's ex-ante costs in that period as follows:

$$
\begin{aligned}
C_i^P(h,\delta_i|\sigma) &= \underbrace{qb^2}_{\text{Cost of bias}} + \underbrace{\sum_{m\in\{0,1\}} \Pr(m)\lambda(m)(1-\lambda(m))}_{\text{Cost of miscommunication}} \\
C_i^A(h,\delta_i|\sigma) &= \underbrace{(1-q)b^2}_{\text{Cost of bias}} + \underbrace{\sum_{m\in\{0,1\}} \Pr(m)\lambda(m)(1-\lambda(m))}_{\text{Cost of miscommunication}}
\end{aligned}
$$

This cost is composed of two components: The first component (*cost of bias*) comes from the fact that the agent plays the biased action with probability q after both messages. The second component (*cost of miscommunication*) comes from the fact that the principal may not provide full information. If, for

[13] See Maskin and Tirole (2001) for the notion of payoff irrelevant histories and some arguments in favor of focusing on Markov perfect equilibria.

[14] More precisely, $\Pr(1) = 0.5\mu_i(h,\delta_i)$ and $\Pr(0) = 0.5 + 0.5(1-\mu_i(h,\delta_i))$.

example, the principal's message provides no information on θ_i, then $\lambda(m) = 1/2$ for $m \in \{0,1\}$ and the cost of miscommunication is equal 1/4. If it is perfectly informative, then $\lambda(1) = 1$ and $\lambda(0) = 0$ and the cost of miscommunication is equal to zero.

To fix ideas, we begin our analysis with the simple case of $N = 1$. In this game, sequential rationality implies that the biased agent plays the biased action, i.e., $a^b(\lambda)$ for any belief λ. As it is the case in cheap-talk games, there is always an equilibrium in which the principal's message provides no information about the state; the so called "babbling equilibrium." The following lemma argues that there are only babbling equilibria if $pb > \frac{1}{2}$. It also shows that there are three types of equilibria if $pb \le \frac{1}{2}$: one in which the principal reports the state truthfully; another in which the principal's report is partially truthful; and babbling equilibrium. Define

$$\bar{q} = \frac{1}{2b}$$

and note that $\bar{q} < 1$ if the bias exceeds 1/2.

Lemma 1. *Let $N = 1$ and $p \in (0,1)$ be the probability that the agent is biased. In any equilibrium, the agent plays the biased action with probability one. If $p > \bar{q}$, then the principal's message provides no information. If $p \le \bar{q}$, then there is an equilibrium where the principal truthfully reports the state and if $1/4b < p < \bar{q}$, then there is an equilibrium where the principal's report is partially truthful (i.e., the principal sends the same message with positive probability in both states). These are the only equilibria.*

Proof. Sequential rationality of the agent implies that he plays his best period action in the last period. For proofs of the other parts see the proof of Lemma 2 in Section 7. □

Remark 1. Note that the equilibria described in Lemma 1 are Pareto ranked: the more informative equilibrium yields a strictly higher expected payoff to both the principal and the agent. In fact, in the truthful equilibrium, the agent's cost is equal to zero whereas the (ex-ante) cost of the principal is pb^2. In the partially informative equilibrium, agent's cost is $(1/2 - pb)$, whereas the principal's is equal to $pb^2 + (1/2 - pb)$. In the babbling equilibrium, expected costs of the agent and the principal are 1/4 and $pb^2 + 1/4$, respectively.

5. THE MAIN RESULT

In this section, we will show that there is a unique principal-optimal equilibrium outcome. Furthermore, this outcome is also optimal for the agent as long as the agent has a sufficiently bad initial reputation. In other words, for sufficiently bad initial reputation levels, there is a unique equilibrium outcome that Pareto dominates all other equilibrium outcomes.

Focusing on the principal-optimal equilibrium, we show that the optimal career path of the agent is characterized by progressively more important decisions if the agent's bias exceeds 1/2. Also, as the agent's bias increases, the initial decision becomes less important but promotion takes place faster. If, on the other hand, the agent's bias is less than 1/2, then the importance of the decisions decreases over time, and as the agent's bias increases, the initial decision becomes less important while the importance of the decisions decreases at a slower rate.

In order to facilitate the definition of the principal-optimal equilibrium we first need some preliminary definitions. Let $\delta_1^* = 0$ and define δ_i^* recursively as

$$\delta_2^* = \frac{4b^2}{1+4b^2} \tag{5.1}$$

$$\delta_i^* = \frac{4b^2}{1+4b^2\prod_{j=2}^{i-1}\delta_j^*},\ i = 3,\dots,N \tag{5.2}$$

For any $p \in [0,1]$, let[15]

$$q_i^*(p) = \begin{cases} \left(1 - \frac{1-p}{(1-\overline{q})^{i-1}}\right)^+, & \overline{q} < 1 \text{ or } i = 1 \\ 0, & \text{otherwise} \end{cases} \tag{5.3}$$

Define the set of period i histories H_i^* as follows. $H_N^* = \{\emptyset\}$ and, for any $i = N-1,\dots,1$, a history $h = (o_N, o_{N-1}\dots, o_{i+1}) \in H_i^*$ if (1) period N outcome, $o_N = (\delta_N, \theta_N, m_N, a_N)$, is such that $\delta_N = \delta_N^*$ and

$$a_N = \begin{cases} m_N, & \text{if } q_N^*(p) \le \overline{q} \\ 1/2, & \text{if } q_N^*(p) > \overline{q} \end{cases}$$

(2) for all $j = N-1,\dots,i+1$, period j outcome, $o_j = \left(\delta_j, \theta_j, m_j, a_j\right)$, is such that $\delta_j = \delta_j^*$ and $a_j = m_j$. In other words, a history belongs to H_i^* if in each previous period j the principal chooses δ_j^* and the agent plays the unbiased action after each message believing that the principal is telling the truth (except in period N, where she believes the principal is telling the truth if and only if doing so is sequentially rational for the principal).

We define the assessment $\sigma^* = \left(\tau_i, \mu_i, q_i, p_i, \lambda_i\right)$ as follows. After each history in H_i^*, the principal chooses δ_i^* and after any other history he chooses $\delta_i = 0$, i.e.,

$$\tau_i(h) = \begin{cases} \delta_i^*, & h \in H_i^* \\ 0, & \text{otherwise} \end{cases} \tag{5.4}$$

If $h \in H_i^*$ and the principal has chosen δ_i^* in period i, then the agent's total probability of playing the biased action is equal to $q_i^*\left(p_i(h)\right)$, where q_i^* is defined in (5.3); otherwise, the biased agent plays the biased action with probability one, i.e.,[16]

$$q_i(h,\delta_i) = \begin{cases} q_i^*\left(p_i(h)\right), & \text{if } h \in H_i^*, \delta_i = \delta_i^*, p_i(h) < 1 \\ p_i(h), & \text{otherwise} \end{cases} \tag{5.5}$$

The principal communicates truthfully in period i as long as the total probability of the biased action in that period is less than or equal to $\overline{q}$, the history belongs to H_i^*, and he has chosen δ_i^*. Since we assumed type $\theta_i = 0$ principal sends message $m_i = 0$, this implies that the probability with which type $\theta_i = 1$ send message $m_i = 1$ is given by

$$\mu_i(h,\delta_i) = \begin{cases} 1, & \text{if } h \in H_i^*, \delta_i = \delta_i^*, q_i(h,\delta_i) \le \overline{q}, p_i(h) < 1 \\ 0, & \text{otherwise} \end{cases} \tag{5.6}$$

[15] $x^+ = \max\{0, x\}$ for any $x \in \mathbb{R}$.

[16] Note that in the last period $\delta_1 = \delta_1^* = 0$ by construction.

In any period i, the unbiased action is given by $a^0(\lambda_i(h,\delta_i,m))$, where

$$\lambda_i(h,\delta_i,m) = \begin{cases} \frac{1-\mu_i(h,\delta_i)}{2-\mu_i(h,\delta_i)}, & m=0 \\ 1, & m=1 \end{cases} \tag{5.7}$$

Beliefs on the agent's type are defined as follows: $p_N(\emptyset) = p$ and

$$p_{i-1}(h,o_i) = 1 - \frac{1-p_i(h)}{1-q_i(h,\delta_i)}, \text{ for all } h \in H_i,\ o_i \in O_i \tag{5.8}$$

Note that if the players play according to σ^* up to and including period $i+1$, which implies that $h_i \in H_i^*$, and the principal chooses $\delta_i = \delta_i^*$, then $h_{i-1} \notin H_{i-1}^*$ if and only if in period i the agent plays an action that is different from the unbiased action, i.e., plays $a_i \neq m_i$. In that case, the principal assigns probability one to the event that the agent is biased, provides no information, and terminates the game by choosing $\delta_{i-1} = 0$.

Theorem 1. *The assessment σ^* is a perfect Bayesian equilibrium and induces the unique principal-optimal equilibrium outcome. If $p > 1-(1-\bar{q})^{N-1}$, then σ^* is also an agent-optimal equilibrium.*

In the equilibrium σ^* described above, the principal leaves a proportion δ_i^* of decisions to subsequent periods on the equilibrium path. This proportion leaves the agent exactly indifferent between the biased and the unbiased action in period i if, in each subsequent period, the principal communicates truthfully after observing the unbiased action in all prior periods and provides no information otherwise.

In order to further describe the equilibrium, we need to consider two cases: Case (1) The agent has a good initial reputation, i.e., $p \le \bar{q}$. In this case, truthful communication is possible even when $N = 1$; Case (2) The agent has a bad initial reputation, i.e., $p > \bar{q}$. In this case, truthful communication is not possible in the one-shot game, but in the repeated game that we consider, it is possible in all periods except possibly the first period, as we will explain further below.

First, suppose that the agent has a good reputation, i.e., $p \le \bar{q}$. In this case, on the equilibrium path, the agent plays the unbiased action in every period except the last one and the principal communicates truthfully in every period.

Instead suppose that the agent has a bad reputation, i.e., $p > \bar{q}$. We can describe equilibrium behavior more precisely by focusing on two sub-cases: (1) N is sufficiently large so that $p \le 1-(1-\bar{q})^{N-1}$ and (2) N is small so that $p > 1-(1-\bar{q})^{N-1}$.

If N is sufficiently large, then the agent plays the unbiased action with probability one until the game reaches period k, where k is the first period (largest integer) such that $p > 1-(1-\bar{q})^{k-1}$. The agent plays the biased action with total probability equal to $1-(1-p)/(1-\bar{q})^{k-1} \le \bar{q}$ in period k and plays the biased action with total probability $\bar{q}$ in all subsequent periods. The agent's reputation remains constant and equal to $1-p$ until the game reaches period k and then monotonically increases in each period to reach exactly $1-\bar{q}$ in the last period of the game. The principal reports the state truthfully in every period after observing the unbiased action.

If, on the other hand, N is small, then the agent plays the biased action with total probability equal to $1-(1-p)/(1-\bar{q})^{N-1}$ in period N and plays the biased action with total probability $\bar{q}$ thereafter. The principal reports the state truthfully in every period except possibly the first period. In the first period, total probability of the biased action may exceed $\bar{q}$ and if this is the case the principal communicates no information. In other words, if the agent has a sufficiently bad initial reputation, then informative communication may fail in the first period but communication is fully informative thereafter.

Figure 5.1 plots the importance parameter (γ_i), reputation of the agent ($1-p_i$), and the total probability with which the agent plays the biased action (q_i) for each period i, when the bias is equal to 1, the prior on b is 0.9, and total number of periods is 10. Note that $\overline{q} = 1/2$ and hence $k = 4$.

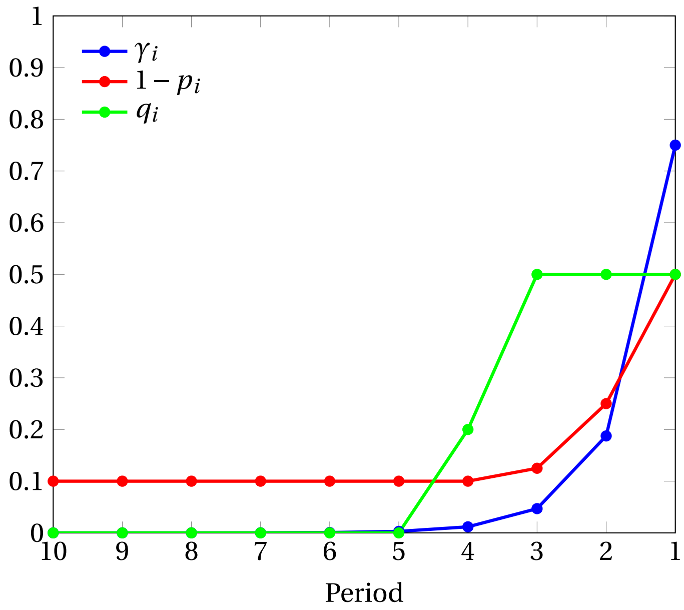

Figure 5.1: $b = 1, p = 0.9$

5.1. The two-period model. In this subsection, we provide some intuition for Theorem 1 by analyzing the simpler two-period version of the model. We will show that the strategy profile σ^* is an equilibrium and the outcome of this strategy profile is the unique principal-optimal equilibrium outcome. Moreover, if $p > \bar{q}$, then this is also the agent-optimal equilibrium outcome.

We first argue that σ^* is an equilibrium. Under σ^*, total probability of the biased action in the last period is $q_1(h, \delta_1) = p_1(h)$ for any history h (see (5.3) and (5.5)). In other words, the biased agent plays the biased action with probability one in the last period, which is sequentially rational. The principal reports truthfully if and only if $p_1(h) \leq \overline{q}$ and $h \in H_1^*$ (see (5.6)), which is sequentially rational for the principal (see Lemma 1). It is easy to check that beliefs λ_1 and p_1, defined in (5.7) and (5.8), respectively, satisfy the Bayes' rule.

Now let us consider the first period behavior. Note that a history is in H_1^* if and only if in the first period the principal has chosen δ_2^* and the agent has played the unbiased action given her beliefs. The agent plays the biased action with total probability $q_2^*(p)$ if $\delta_2 = \delta_2^*$ and with total probability p otherwise (see (5.5)). If $\delta_2 = \delta_2^*$ and the agent plays the unbiased action, then she induces a history that belongs to H_1^* and consequently learns the state perfectly in the next period and plays her best period action. This implies that the total cost of playing the unbiased action is $(1-\delta_2^*)b^2$. If the agent plays the biased action, then her cost is equal to zero in the current period, but she induces a history that is not in H_1^* and receives no information in the next period, which costs her $1/4$. Therefore, the total cost of playing the biased action is equal to $\delta_2^*/4$. Definition of δ_2^* (see (5.1)) implies that $(1-\delta_2^*)b^2 = \delta_2^*/4$, i.e., the agent is indifferent between the biased and the unbiased actions and hence playing the biased action with total probability $q_2^*(p)$ is sequentially rational. If, on the other hand, $\delta_2 \neq \delta_2^*$, then the history is not in H_1^* irrespective of what the agent does, which implies that the principal will provide no information in the next period (see (5.6)). Therefore, it is sequentially rational to play the biased action with probability one after any $\delta_2 \neq \delta_2^*$. Finally, given the behavior of the agent in the first period, communication strategy of the principal specified in (5.6) is sequentially rational.[17] Therefore, we conclude that σ^* is a perfect Bayesian equilibrium.

We will now show that σ^* yields the unique principal-optimal equilibrium outcome. Fix $p \in (0,1)$ and consider an equilibrium where the principal chooses $\delta_2 \in [0,1)$ and the agent plays the biased action with total probability $q_2 \in [0,p]$ in the first period after both messages. The principal's ex ante total cost in such an equilibrium is

$$\begin{aligned} C\left(\delta_2, q_2, c_2, c_1^b, c_1^u\right) &= (1-\delta_2)\underbrace{\left[q_2 b^2 + c_2\right]}_{\text{Period 2 cost}} + \delta_2 [q_2 \underbrace{(b^2 + c_1^b)}_{\text{Cost after biased action}} + (1-q_2)\underbrace{(p_1 b^2 + c_1^u)}_{\text{Cost after unbiased action}}] \\ &= (1-\delta_2)\left[q_2 b^2 + c_2\right] + \delta_2\left[p b^2 + q_2 c_1^b + (1-q_2) c_1^u\right] \end{aligned} \tag{5.9}$$

where $p_1 = (p-q_2)/(1-q_2)$ by Bayes' rule, the cost of miscommunication in period 1 is equal to c_1^b and c_1^u after the biased and unbiased actions, respectively, and c_2 is the cost of miscommunication in period 2. For notational simplicity let $q_2^* = q_2^*(p)$ and note that $q_2^* < p$. The cost under σ^* is equal to

$$C^* = C\left(\delta_2^*, q_2^*, c_2^*, 0, 0\right) = (1-\delta_2^*)\left[q_2^* b^2 + c_2^*\right] + \delta_2^*\left[p b^2 + \frac{q_2^*}{4}\right]$$

where $c_2^* = 0$ if $q_2^* \le \bar{q}$ and $c_2^* = 1/4$ otherwise.

Assume first that $\delta_2 < \delta_2^*$. This implies that the biased agent plays the biased action with probability one in the first period, i.e., $q_2 = p$. This is because, the cost of playing the biased action in the first period is at most $\delta_2/4$, while the cost of the unbiased action is at least $(1-\delta_2)\,b^2$ and $\delta_2 < \delta_2^*$ implies $\delta_2/4 < (1-\delta_2)\,b^2$. The principal's cost in such an equilibrium is

$$C' = C\left(\delta_2, p, c_2, c_1^b, c_1^u\right) = p b^2 + (1-\delta_2)c_2 + \delta_2\left(p c_1^b + (1-p)\,c_1^u\right)$$

Therefore,

$$C' - C^* = b^2(1-\delta_2^*)(p-q_2^*) + (1-\delta_2)c_2 + \delta_2\left(p c_1^b + (1-p)\,c_1^u\right) - (1-\delta_2^*)c_2^* - \delta_2^*\frac{q_2^*}{4}$$

If $p \le \bar{q}$, then $q_2^* = c_2^* = 0$, which implies that $C' - C^* > 0$. If $p > \bar{q}$, then $b > 1/2$ and Lemma 2 implies

[17] The key to this observation is the fact that under σ^*, the principal's continuation payoff depends only on whether $\delta_2 = \delta_2^*$ and whether the agent plays the unbiased action. In other words, his continuation payoff does not depend on θ_2 or m_2, which implies that it is sequentially rational to provide full information if and only if the total probability of the biased action in the first period is smaller than or equal to $\overline{q}$. See Lemma 2 in Section 7 for the details of this argument.

that $c_2 = c_1^b = 1/4$. Therefore,

$$\begin{aligned} C' - C^* &= b^2(1-\delta_2^*)(p-q_2^*) + (1-\delta_2)\frac{1}{4} + \delta_2\left(p\frac{1}{4} + (1-p)\,c_1^u\right) - (1-\delta_2^*)c_2^* - \delta_2^*\frac{q_2^*}{4} \\ &\geq b^2(1-\delta_2^*)(p-q_2^*) + \frac{1}{4}\left(\delta_2^*\left(1-q_2^*\right) - \delta_2\left(1-p\right)\right) \\ &> b^2(1-\delta_2^*)(p-q_2^*) + \frac{1}{4}\delta_2^*\left(p-q_2^*\right) \\ &> 0 \end{aligned}$$

where the first inequality follows from $c_2^* \leq 1/4$, the second from $\delta_2 < \delta_2^*$, and the third from $q_2^* < p$. Therefore, the cost in any equilibrium in which $\delta_2 < \delta_2^*$ is strictly greater than the cost in equilibrium σ^*.

Assume now that $\delta_2 \geq \delta_2^*$. We will consider two cases separately: (1) $p \leq \bar{q}$ and (2) $p > \bar{q}$. Assume first that $p \leq \bar{q}$, i.e., the agent has a good reputation. In this case, $q_2^* = c_2^* = 0$ and the principal's cost under σ^* is $C^* = \delta_2^* p b^2$. The principal's cost in any equilibrium with $\delta_2 \geq \delta_2^*$ and $q_2 > 0$, on the other hand, is at least $C' = (1-\delta_2)q_2 b^2 + \delta_2 p b^2$, which is strictly greater than $\delta_2^* p b^2$ for any $\delta_2 \geq \delta_2^*$ and $q_2 > 0$. Intuitively, under strategy profile σ^*, the principal minimizes the amount of decisions left for the last period (where the biased agent plays the biased action with probability one) subject to the constraint $\delta_2 \geq \delta_2^*$, which provides incentives to the biased agent to choose the unbiased action in the first period.

Assume now that $p > \bar{q}$, i.e., the agent has a bad reputation, and note that this implies $b > 1/2$. In this case, there is no equilibrium where the total probability of the biased action in the first period is less than q_2^*. In order to establish this, suppose, to the contrary, that there is an equilibrium where $q_2 < q_2^*$. Bayes' rule implies that, following the unbiased action, the agent's reputation next period is equal to $1 - p_1 = \frac{1-p}{1-q_2}$. Therefore, our definition of $q_2^* = 1-(1-p)/(1-\bar{q})$ and $q_2 < q_2^*$ imply that $p_1 > \bar{q}$. However, if $p_1 > \bar{q}$, then the principal will not provide any information in the last period (see Lemma 1). But if the principal provides no information in the last period even after observing the unbiased action, then the biased agent has no incentive to play the unbiased action in the first period: doing so does not change her expected payoff in the subsequent period and decreases her payoff in the current period. We conclude that the biased agent plays the biased action with probability one in the first period, which contradicts the hypothesis that $q_2 < q_2^* \leq p$.

The argument in the previous paragraph implies that $q_2 \geq q_2^*$ in any equilibrium where $\delta_2 \geq \delta_2^*$ and $p > \bar{q}$. Also, $b > 1/2$ implies that the cost of miscommunication after the biased action is $c_1^b = 1/4$. The principal's cost in such an equilibrium is

$$C\left(\delta_2, q_2, c_2, \frac{1}{4}, c_1^u\right) = (1-\delta_2)\left[q_2 b^2 + c_2\right] + \delta_2\left[p b^2 + q_2\frac{1}{4} + (1-q_2)c_1^u\right].$$

Note that

$$C\left(\delta_2, q_2, c_2, \frac{1}{4}, c_1^u\right) \geq C\left(\delta_2, q_2, c_2, \frac{1}{4}, 0\right)$$

since deleting the communication costs in the last period will only decrease the principal's cost. Also, note that the function

$$C\left(\delta_2, q_2, c_2, \frac{1}{4}, 0\right) = (1-\delta_2)\left[q_2 b^2 + c_2\right] + \delta_2\left[p b^2 + q_2\frac{1}{4}\right]$$

is strictly increasing in q_2: the principal would rather have the agent play the biased action later rather than sooner. Therefore, we find that

$$C\left(\delta_2, q_2, c_2, \frac{1}{4}, c_1^u\right) \geq C\left(\delta_2, q_2, c_2, \frac{1}{4}, 0\right) \geq C\left(\delta_2, q_2^*, c_2, \frac{1}{4}, 0\right)$$

because $q_2^* \leq q_2$. Also, $C\left(\delta_2, q_2^*, c_2, \frac{1}{4}, 0\right) \geq C\left(\delta_2, q_2^*, c_2^*, \frac{1}{4}, 0\right)$ because $q_2^* \leq q_2$ and Lemma 2 imply that $c_2^* \leq c_2$. Direct computation shows that $C\left(\delta_2, q_2^*, c_2^*, \frac{1}{4}, 0\right)$ is strictly increasing in δ_2:

$$\begin{aligned}
\frac{\partial}{\partial \delta_2} C\left(\delta_2, q_2^*, c_2^*, \frac{1}{4}, 0\right) &\geq \left(p - q_2^*\right) b^2 - \left(1 - q_2^*\right) \frac{1}{4} \\
&= \left(1 - q_2^*\right) \overline{q} b^2 - \left(1 - q_2^*\right) \frac{1}{4} \\
&= \left(1 - q_2^*\right) \left(\frac{b}{2} - \frac{1}{4}\right) \\
&> 0.
\end{aligned}$$

Intuitively, the principal would prefer not to leave too much to the last period, where the biased agent plays the biased action with probability one. Therefore, $C\left(\delta_2, q_2^*, c_2^*, \frac{1}{4}, 0\right) \geq C\left(\delta_2^*, q_2^*, c_2^*, \frac{1}{4}, 0\right)$ for any $\delta_2 \geq \delta_2^*$ and hence $C\left(\delta_2, q_2, c_2, \frac{1}{4}, c_1^u\right) \geq C\left(\delta_2^*, q_2^*, c_2^*, \frac{1}{4}, 0\right)$. Therefore, the cost in any equilibrium where $\delta_2 \geq \delta_2^*$ is greater than or equal to the cost in equilibrium σ^*. This proves that σ^* leads to the minimum possible cost for the principal. Furthermore, in any equilibrium that satisfies Property 2 such that $\left(\delta_2, q_2, c_2, c_1^u\right) \neq \left(\delta_2^*, q_2^*, c_2^*, 0\right)$, the principal's cost is strictly higher, which implies that σ^* leads to the unique principal-optimal equilibrium outcome.

Note that in this equilibrium, the principal leaves more important decisions to the last period, i.e., $\delta_2^* > 1/2$, if and only if $b > 1/2$, i.e., the bias is large enough to make truthful communication with the biased agent impossible in the one-shot game.

We now argue that σ^* also yields the highest equilibrium payoff for the agent if $p > \bar{q}$. Note that the agent's total cost under σ^* is $\left(1 - \delta_2^*\right) c_2^* + \delta_2^*/4$, where $c_2^* = 0$ if $q_2^* \leq \overline{q}$, and $c_2^* = 1/4$ otherwise. As we have argued above, if $\delta_2 < \delta_2^*$, then the biased agent plays the biased action with probability one in the first period. Lemma 2, together with $p > \overline{q}$, implies that the agent's cost is $1/4$, which is greater than or equal to her cost under σ^*. Therefore, in any agent-optimal equilibrium we must have $\delta_2 \geq \delta_2^*$. We have already shown that $p > \bar{q}$ implies that there is no equilibrium where $q_2 < q_2^*$. If $q_2 \geq q_2^*$, then the agent's total cost is equal to $(1 - \delta_2) c_2 + \delta_2/4$ and there are two cases to consider: (1) $q_2^* \leq \overline{q}$, in which case the cost in the principal-optimal equilibrium is smaller: $\delta_2^*/4 \leq (1 - \delta_2) c_2 + \delta_2/4$; (2) $q_2^* > \overline{q}$, in which case the cost in both cases is $1/4$. Therefore, there is no equilibrium that yields the agent a higher payoff when $p > \bar{q}$.[18]

To summarize, in any principal-optimal equilibrium: (1) The principal chooses the relative importance of the first period so that the agent is indifferent between the biased and the unbiased actions for that period, given that the principal will communicate truthfully after the unbiased action and will provide no information otherwise; (2) If the initial reputation of the agent is good, then she plays the unbiased action in the first period, while if it is bad, she mixes in such way that her reputation next period is just good enough to make truthful communication possible; (3) The principal

[18] If $p \leq \overline{q}$, then the best equilibrium outcome for the agent is to play the biased action and receive full information in both periods, which is different from the principal-optimal equilibrium outcome.

communicates truthfully in both periods if and only if the agent has a sufficiently good initial reputation; (4) The principal leaves more important decisions to the future if and only if the bias is large enough.

5.2. Sequencing of Decisions. We are now ready to answer the main question that motivated our model: How would the principal choose the importance of the decisions he delegates to the agent? Or equivalently, what is the optimal career path of the agent from the perspective of the principal?

Proposition 1. *Suppose that play unfolds according to equilibrium σ^* and denote by γ_i^* the importance of decisions made in period i in this equilibrium. If $b > 1/2$, then the optimal career path of the agent is characterized by progressively more important decisions, i.e., $\gamma_N^* < \gamma_{N-1}^* < \cdots < \gamma_2^* < \gamma_1^*$, whereas if $b < 1/2$, then the importance of the decisions decreases over time, i.e., $\gamma_N^* > \gamma_{N-1}^* > \cdots > \gamma_2^* > \gamma_1^*$. As b increases, the initial decision becomes less important; but the growth rate of the importance of decisions increases, i.e., γ_j^*/γ_i^* is increasing in b for all $j < i$.*

Proof. See Section 7. □

The proof of Proposition 1 shows that the unique solution for δ_i^* is given by

$$\delta_i^* = \frac{\sum_{j=1}^{i-1} a^j}{\sum_{j=0}^{i-1} a^j},$$

where $a = 4b^2$. We can then solve for the importance parameters as

$$\gamma_{N-i} = \frac{a^i}{\sum_{j=0}^{N-1} a^j}, \; i = 0, 1, \ldots, N-1.$$

In other words, the weight of the first decision, i.e., decision N, is given by

$$\gamma_N = \frac{1}{\sum_{j=0}^{N-1} a^j},$$

and then each subsequent weight is just a times the previous one. If $a > 1$, i.e., $b > 1/2$, this implies that each period receives more weight than the previous one. More precisely, the growth rate of the importance parameter is equal to $\ln a > 0$, i.e., the greater the potential bias of the agent, the higher the growth rate of the importance of decisions delegated, or equivalently the faster the agent is promoted. If, on the other hand $a < 1$, i.e., $b < 1/2$, then the importance of the decisions decreases over time.

Figure 5.2 plots the evolution of the importance parameter over time for three different bias parameters, two of which are greater than 1/2 and one is smaller than 1/2. Observe that, when the potential bias is large, the principal delegates mostly trivial tasks in the beginning, but promotes the agent very fast towards the end of her career.

Finally, we can show that the equilibrium cost of the principal is strictly decreasing in the number of periods N.

Proposition 2. *Total cost of the principal strictly decreases in N and it has a strictly positive lower bound if and only if $b > 1/2$.*

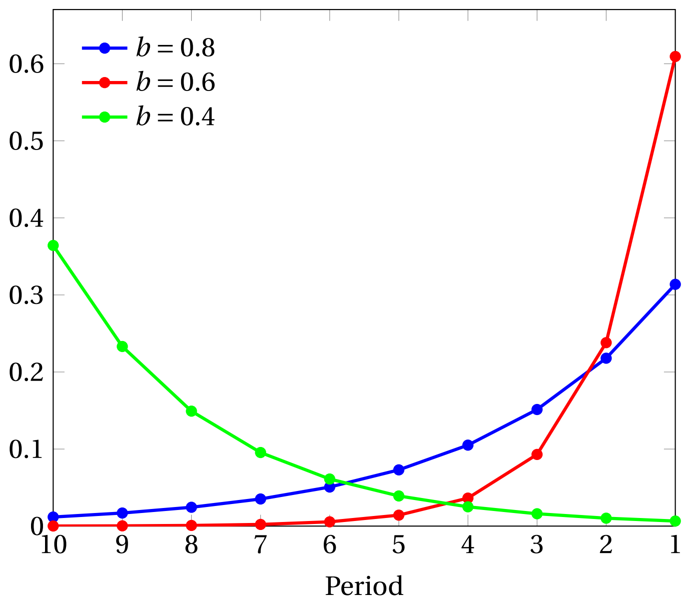

Figure 5.2: Importance parameter for different biases

Proof. See Section 7. □

This result implies that, if the principal had a choice over the number of periods over which to spread the decisions, then she would choose as many periods as possible. Of course, this neglects any cost of time, which would act as a countervailing force. Secondly, this result shows that if the potential conflict of interest is large, then there is a lower bound to the cost of delegation, i.e., delegation is always costly.

6. Concluding Remarks

We have analyzed a model in which an informed principal delegates a set of decisions over time to an uninformed and potentially biased agent. We find that, if the potential bias is large, then the principal progressively increases the importance of decisions she assigns to the agent. In other words, the agent starts her career at a lower rank in the hierarchy and is promoted as long as she does not make decisions that reveal her as a biased agent. Furthermore, the larger the potential bias of the agent, the lower the initial rank and faster the promotion.

Basically, the principal designs the career path of the agent in order to exploit her incentives to build reputation for being unbiased. In equilibrium, the agent plays the principal's favorite action in the beginning of her career while towards the end she takes risks by playing her own favorite action with positive probability. Principal's optimal design also allows him to communicate truthfully with the agent throughout their relationship as long as she always does the "right thing."

We also showed that if the potential bias is small, i.e., smaller than $1/2$, then this pattern is reversed and the importance of the decisions decreases over time. Note that if $b < 1/2$, or more generally the initial reputation of the agent is good enough to make truthful communication possible even in the one-shot game, i.e., $pb < 1/2$, the principal-optimal equilibrium is not agent-optimal. In fact, in this case, equilibrium is sustained by the principal's off-the-equilibrium threat to communicate no information if the agent plays the biased action. This threat is perfectly credible when $b > 1/2$, because the only equilibrium behavior once the agent is revealed to be biased is to reveal no information. The same is not true when $b < 1/2$. Since the principal prefers truthful communication ex-ante, such a threat may be regarded as non-credible. If that is the position one takes, then our results should be deemed most convincing and interesting for those cases in which the potential bias of the agent is large enough.

The current work raises many other questions and could be extended in a number of ways. For example, what would be the equilibrium allocation of decisions in a situation where reputational concerns create perverse incentives as in Morris (2001), Jeffrey C. Ely (2003), Maskin and Tirole (2004), or Kartik and Van Weelden (2015)? As opposed to what happens in our model, would it be optimal to front-load the decisions in order to avoid such perverse incentives? More technical extensions include richer type spaces for the players, but our preliminary analyses of such models have so far proved non-trivial.

Finally, although we assumed that the unbiased agent is a commitment type who plays her myopic best response in each period, this behavior can be sustained as an equilibrium of a model in which she is a fully strategic player. In fact, such a behavior seems intuitively the most plausible, disregarding equilibria where she plays an extreme action in order to separate herself from the biased agent. We did not consider such equilibria, which raise questions similar to the ones raised by the "bad reputation" models mentioned in the previous paragraph.

7. Proofs

Proof. [Proof of Theorem 1] For any assessment σ, period $i \in \{1,\dots,N\}$, and history $h \in H_i$, let $V_i(\sigma|h)$ be the expected continuation payoff of the principal under σ conditional on reaching h and define $V_0(\sigma|h) = 0$. Property 2 implies that in equilibrium, the continuation payoff of the principal is constant over the state and the message, i.e., for any $h \in H_i$, $V_{i-1}(\sigma|h,\delta,\theta,m)$ is constant over $(\theta,m) \in \{0,1\}^2$. As the following lemma shows, this implies that the equilibrium communication behavior of the principal in any period depends only on the total probability of the biased action in that period only. As in all cheap-talk games, non-informative communication always exists. Whether informative communication is possible depends on the probability with which the agent plays the biased action.

Lemma 2. *Fix a perfect Bayesian equilibrium σ, a period $i \in \{1,\dots,N\}$, history $h \in H_i$, and $\delta \in [0,1]$. Suppose that the total probability with which the agent plays the biased action is $q_i(h,\delta) = q$ after each message. Then the principal's equilibrium communication strategy can be of three types: (1) fully informative; (2) partially informative; or (3) non-informative. It is completely informative only if $qb \leq 1/2$ and partially informative only if $1/4 < qb < 1/2$.*

Proof. [Proof of Lemma 2] Let $\lambda_m = \lambda_i(h,\delta,m)$ be the equilibrium belief after message m and note that the period cost of sending message m for type $\theta \in \{0,1\}$ is

$$(\lambda_m - \theta)^2 + 2(\lambda_m - \theta)qb + qb^2.$$

Since the continuation payoff is invariant under the state and the message, only the period payoff matters for sequential rationality of the principal. As it is always the case in cheap-talk models, there is always an equilibrium in which the principal's strategy is completely uninformative irrespective of his beliefs, the so called "babbling equilibrium." Suppose that in equilibrium the principal provides full information to the agent. Sequential rationality of type $\theta_i = 0$ is always satisfied, whereas sequential rationality of type $\theta_i = 1$ implies that $qb^2 \leq 1-2qb+qb^2$ or $qb \leq 1/2$. If type $\theta_i = 0$ plays a completely mixed strategy, then $\lambda_1^2 + 2\lambda_1 qb = \lambda_0^2 + 2\lambda_0 qb$, which implies $\lambda_0 = \lambda_1$. This implies that both types mix with equal probabilities and hence the principal's strategy is non-informative. Therefore, in any other type of equilibrium behavior, type 0 must be playing a pure strategy while type 1 completely mixes. Suppose, without loss of generality, that type 0 send message 0. This implies that $\lambda_1 = 1$ and $\lambda_0 \in (0, 1/2)$. It is easy to show that type 0's sequential rationality is satisfied while type 1's sequential rationality implies that $\lambda_0 = 1-2qb$, which, in turn, implies that $1/4 < qb < 1/2$. □

Lemma 3. *The assessment σ^* is a perfect Bayesian equilibrium.*

Proof. Fix a history $h \in H_1^*$ and note that under σ^* the biased agent plays the biased action with probability one and the principal provides information if and only if $p_1(h) \leq \overline{q}$. Agent's strategy is sequentially rational since period 1 is the last period and the principal's strategy is sequentially rational by Lemma 2.

Let $i > 1$ and fix a history $h_i \in H_i^*$ such that $p_i(h_i) < 1$. If $\delta_i = \delta_i^*$, then under σ^* the agent is indifferent between the biased and unbiased actions after any message m_i. In order to see this, note that it is true for $i = 2$, as we previously showed in section 5.1. Suppose that it is true in period $i-1$. If, in period i, the agent chooses the biased action, then she induces a history that is not in H_{i-1}^*, which implies that in period $i-1$ the principal chooses $\delta_{i-1} = 0$, provides no information, and the agent plays the biased action. Therefore, the cost of playing the biased action is $\delta_i^*/4$. If she plays the unbiased action instead, then she suffers a cost equal to b^2 in period i but induces a history in H_{i-1}^*. In the next period, the principal choses δ_{i-1}^* and provides full information. Under the induction hypothesis, her expected cost starting from period $i-1$ is equal to $\delta_{i-1}^*/4$, i.e., the cost of playing the biased action in period $i-1$. Therefore, the cost of playing the unbiased action is period i is equal to $\left(1-\delta_i^*\right)b^2 + \delta_i^*\delta_{i-1}^*/4$. Definition of δ_i^* (see 5.2) implies that

$$\delta_i^* \frac{1}{4} = \left(1-\delta_i^*\right)b^2 + \delta_i^*\delta_{i-1}^*\frac{1}{4}$$

which, in turn, implies that she is indifferent between the biased and unbiased actions in period i. Therefore, playing the biased action with total probability $q_i^*\left(p_i(h)\right)$ is optimal after such histories.

Lemma 2 implies that the communication strategy of the principal (see (5.6)) is sequentially rational because after any $h \in H_i^*$ and δ_i^*, the principal's continuation payoff is constant over $(\theta_i, m_i) \in \{0,1\}^2$.If the principal chooses $\delta_i \neq \delta_i^*$, then in period i he provides no information and the biased agent plays the biased action. In period $i-1$, he chooses $\delta_{i-1} = 0$, provides no information, and the biased agent again plays the biased action. Therefore, his expected cost of choosing $\delta_i \neq \delta_i^*$ is equal to

$$p_i(h)\,b^2 + \frac{1}{4}.$$

If he chooses δ_i^*, then the agent plays the biased action with total probability $q_i^*\left(p_i(h)\right) \leq p_i(h)$, and this cannot lead to a higher expected cost. Therefore, it is sequentially rational for the principal to choose δ_i^*.

If $h_i \notin H_i^*$ or $p_i(h) = 1$, then the biased agent plays the biased action with probability one. This is sequentially rational because the principal provides no information in any subsequent period. The principal is willing to provide no information because babbling is always an equilibrium of the cheap-talk game. Moreover, it is sequentially rational for the principal to choose $\delta_i = 0$ because his continuation payoff is equal to $p_i(h_i)b^2 + \frac{1}{4}$ and independent of his choice of δ_i.

Finally, it is straightforward to check that the beliefs defined in (5.7) and (5.8) satisfy the Bayes' rule whenever it can be applied conditional on reaching any $h \in H_i$. □

Lemma 4. *The assessment σ^* is principal optimal.*

Proof. Fix an N period game and assessment σ.

Step 1. If σ is principal optimal, then the total /probability that the agent plays the biased action is at most $\bar{q}$ for all $i < N$ after any history where only the unbiased action has been observed.

First note that this is satisfied automatically if $b < 1/2$. Suppose that there is a period $i < N$ where the agent plays the bias action with strictly higher probability than $\bar{q}$, then the agent plays the biased action with probability $q_j = p > \bar{q}$ in period N.

To see this note that in period $i+1$ the agent's cost from playing the biased action is $\frac{\delta_i}{4} + (1-\delta_i)x$. The agent's cost from playing the unbiased is $(1-\delta_i)(x+b^2) + \frac{\delta_i}{4}$ where the continuation payoff is $\frac{\delta_i}{4}$ because the principal's communication is uninformative in period $i-1$ and the agent plays the biased action with positive probability. Therefore, the biased agent will play the biased action with probability one in period $i+1$. Working this way recursively we find that the agent will play the biased action with probability one in each period $j > i$, i.e., $q_j = p_j$.

The assessment σ^* entails strictly lower cost than σ. This is because the cost under σ is at least

$$C^P(\sigma) \geq p(\frac{1}{4} + b^2).$$

In contrast, the cost in assessment σ^* is at most

$$C^P(\sigma^*) \quad \leq \quad q(b^2 + \frac{1}{4}) + (1-q)((1-\delta)\frac{1}{4} + \delta\frac{p-q}{1-q}b^2)$$

where $\frac{p-q}{1-q} = \bar{q}$. Hence,

$$\begin{aligned} p(\frac{1}{4} + b^2) - C^P(\sigma^*) &\geq (p-q)(b^2 + \frac{1}{4}) - (1-q)((1-\delta)\frac{1}{4} + \delta\frac{p-q}{1-q}b^2) \\ &= (p-q)\frac{1}{4} + (p-q)b^2(1-\delta) - (1-q)((1-\delta)\frac{1}{4} \\ &= (p-q)\frac{1}{4} + \bar{q}(1-q)b^2(1-\delta) - (1-q)(1-\delta)\frac{1}{4} \\ &\geq (p-q)\frac{1}{4} + (\frac{b}{2} - \frac{1}{4})(1-q)(1-\delta) > 0 \end{aligned}$$

Step 2. The miscommunication costs are equal to zero in any period $j < N$.

Suppose that there are information costs x_j in some period $j < N$ in an assessment σ, then there is an assessment σ' that has strictly lower costs for the principal and the agent than σ.

We will use the above assessment, posited to exist, to construct an assessment with no information costs, i.e., one in which the principal communicates truthfully in each period except possibly period N. We will show that this new assessment decreases the principal's cost. Note that we can assume that $q_i \leq \bar{q}$ for all $i < N$ because of Step 1. Therefore, truthful communication is incentive compatible for the principal.

If the principal communicates truthfully in every period, then we have

$$\delta_i \frac{1}{4} > (1 - \delta_i \cdots \delta_2) b^2$$

in any period $i > j$, i.e., playing the unbiased action is strictly preferred by the agent in each period.

Let j be such that there are information costs in this period and no information costs in any period $i \in \{j-1, ..., 1\}$. In any period $i > j$ the following inequality holds

$$\delta_i \frac{1}{4} \quad \geq \quad (1 - \delta_i \cdots \delta_{z+1}) b^2 + \sum_{k=z}^{i-1} (\prod_{l=k+1}^{i} \delta_l)(1 - \delta_k) x_k + \frac{1}{4} \delta_z.$$

where $z < i$ is any period in which the agent plays the biased action with positive probability. Note that such a period must exist because the agent will play the biased action with probability one in the last period of the game. The sum $\sum_{k=z}^{i-1} (\prod_{l=k+1}^{i} \delta_l)(1 - \delta_k) x_k \geq 0$ is the total information costs that the agent incurs in the posited equilibrium in the periods $\{i-1, ..., z\}$. The inequality holds because the agent must prefer to play the unbiased action until period z and then switch to the biased action in period z. Moreover, note that in any period $i > j$ where the agent plays a mixed strategy the inequality holds with equality.

In the equilibrium that we construct, we leave the δ_is unchanged for any period $i \leq j$ and for any period in which the agent plays the unbiased action with probability one.

Let $i > j$ be the first period where the agent plays a mixed strategy and choose $\hat{\delta}_i$ such that

$$\hat{\delta}_i \frac{1}{4} = (1 - \hat{\delta}_i \delta_{i-1} \cdots \delta_2) b^2.$$

Note that $\hat{\delta}_i < \delta_i$. We will now show that for any period $k > i$, the agent strictly prefers to play the unbiased action. For any period $k > i$, the agent would weakly prefer to play the unbiased action with probability 1 until period i, then play the biased action in period i under the posited equilibrium σ. More precisely,

$$\delta_k \frac{1}{4} \geq (1 - \delta_k \cdots \delta_{k-1} \delta_{i+1}) b^2 + X + \delta_i \frac{1}{4}$$

where X denotes the total information costs in periods $\{k-1, ..., i\}$. Therefore, in the equilibrium without information costs we find that,

$$\delta_k \frac{1}{4} > (1 - \delta_k \cdots \delta_{k-1} \delta_{i+1}) b^2 + \hat{\delta}_i \frac{1}{4}$$

Suppose that k is a period where the agent plays a mixed strategy. Suppose that $\delta' s$ are unchanged in any period $i < k$ where the agent plays the unbiased action with probability one, further suppose that $\hat{\delta}_i < \delta_i$ in any period $i < k$ where the agent plays a mixed strategy and suppose that the agent is indifferent in period i given the new choice of δ's. Note that the above argument implies that

$$\delta_k \frac{1}{4} > (1 - \delta_k \cdots \delta_{k-1} \delta_{i+1}) b^2 + \hat{\delta}_i \frac{1}{4}.$$

Pick $\hat{\delta}_k$ such that

$$\hat{\delta}_k \frac{1}{4} = (1-\hat{\delta}_k \cdots \delta_{k-1}\delta_{i+1})b^2 + \hat{\delta}_i \frac{1}{4}$$

and note that $\hat{\delta}_k < \delta_k$. Choosing δs in this manner ensures that the agent is indifferent in any period that he plays a mixed strategy and prefers the unbiased action in any period where he plays the unbiased action with probability one. Also note that $\hat{\delta}_i \le \delta_i$ in any period i.

If we leave everything else the same in the equilibrium σ and decreasing the δ's in the way described above we obtain a new equilibrium where all information costs have been eliminated. Note that in this new equilibrium the principal's costs have strictly decreased. Therefore, the original σ could not have been the principal optimal equilibrium.

Step 3. If $p_j = 1$ for all $j < i$, then $\gamma_j = 0$ for all $j < i$.

Let i be the smallest period where $p_i < 1$. Fix a candidate principal optimal equilibrium σ and assume that $\gamma_j > 0$ for some $j < i$. By the above argument, we know that there are no information costs.

In any period where the agent is mixing we have the following:

$$(1-\delta_k\delta_{k-1}\cdots\delta_{i+1})b^2 + \delta_k\delta_{k-1}\cdots\delta_{i+1}\delta_i\frac{1}{4} = \frac{1}{4}\delta_k.$$

Note that setting δ_i to equal zero is equivalent to deleting an information cost. Therefore, the algorithm outlined in the above step can be used to improve upon this equilibrium.

Step 4. We now complete the argument that the assessment we construct is principal optimal by showing that any principal optimal assessment must solve an optimization problem which we construct below and by showing that the assessment σ^* indeed solves this optimization problem.

Any assessment σ that satisfies steps 1-3 is feasible for the following minimization problem.

$$\min_{q,\gamma} q_1 \prod_{j=2}^{N}(1-q_j)b^2 + \sum_{i=2}^{N}(q_i \prod_{j=i+1}^{N}(1-q_j))(b^2(\sum_{j=1}^{i}\gamma_j) + \frac{1}{4}(\sum_{j=1}^{i-1}\gamma_j))$$

Subject to

$$\begin{aligned}
4b^2\sum_{j=2}^{i}\gamma_j &\le \textstyle\sum_{j=1}^{i-1}\gamma_j \text{ for all } i>1\\
(4b^2\sum_{j=2}^{i}\gamma_j - \sum_{j=1}^{i-1}\gamma_j)q_i &= 0 \text{ for all } i>1\\
q_i &\le \bar{q} \text{ for all } i<N\\
\sum_{i=1}^{N}(q_i\prod_{j=i+1}^{N}(1-q_j)) &\ge p\\
\sum_{j=1}^{N}\gamma_j &= 1\\
\gamma_j &\ge 0\\
q_j &\ge 0
\end{aligned}$$

In the optimization above $\gamma_i = \prod_{j=i+1}^{N}\delta_{j+1}(1-\delta_i)$ and q_i is the total probability of playing the biased action in periods i.

The first IC constraint says that playing the biased action is at least as costly as the unbiased action in every period except the last, i.e.,

$$\frac{\sum_{j=2}^{i}\gamma_j}{\sum_{j=1}^{i}\gamma_j}b^2 \le \frac{\sum_{j=1}^{i-1}\gamma_j}{\sum_{j=1}^{i}\gamma_j}\frac{1}{4}$$

cancelling $\sum_{j=1}^{i}\gamma_j$ from both sides we obtain the constraint. This constraint must hold, because if it did not, then the agent would play the biased action with probability one in that period. However, then the probability of playing the unbiased action would exceed $\bar{q}$ in that period. This would however contradict step 2.

The second constraint says that the first IC constraint can only hold strictly in periods where the agent plays the biased action with probability zero. The third constraint says that the biased type eventually plays the biased action. Therefore, the total probability of the biased action is at least equal to the prior probability that the principal faces a biased agent.

The optimization problem above is feasible because the assessment that we constructed satisfies all of the constraints. Moreover, the constraint set is compact. Therefore, the optimization problem admits a solution. We now argue that our assessment solves the problem.

We argue that the IC constraint $4b^2\sum_{j=2}^{i}\gamma_j \le \sum_{j=1}^{i-1}\gamma_j$ holds with equality for all $i > 1$ in any solution to the optimization problem. Suppose that $4b^2\sum_{j=2}^{i}\gamma_j < \sum_{j=1}^{i-1}\gamma_j$ for some $i < k$. This implies that $q_i = 0$ because of the second constraint.

We show that if we increase γ_i by Δ to $\hat{\gamma}_i$ so that the ith constraint binds, decrease γ_{i+1} by Δ to $\hat{\gamma}_{i+1}$, set $\hat{q}_i = q_{i+1}$, and $\hat{q}_{i+1} = 0$ and leave all other variables unchanged, then all the constraints continue to hold. However, we show that this new feasible choice has strictly lower cost and dominates the old plan.

Let

$$\Delta = (\sum_{j=1}^{i-1}\gamma_j)\frac{1}{4b^2} - (\sum_{j=2}^{i}\gamma_j).$$

This choice of Δ ensures that the ith constraint binds with equality. Note that

$$\begin{aligned}\gamma_{i+1} &= \frac{\gamma_i}{4b^2} + (\sum_{j=1}^{i-1}\gamma_j)\frac{1}{4b^2} - (\sum_{j=2}^{i}\gamma_j)\\ &= \frac{\gamma_i}{4b^2} + \Delta > 0.\end{aligned}$$

Hence, $\hat{\gamma}_{i+1} > 0$. Also, note that the $i+1$st constraint now holds strictly. This is because

$$\begin{aligned}(\hat{\gamma}_{i+1}+\hat{\gamma}_i)b^2 + b^2\sum_{j=2}^{i-1}\gamma_j &= b^2\sum_{j=2}^{i+1}\gamma_j \\ &\le \frac{1}{4}\sum_{j=1}^{i}\gamma_j \\ &= \frac{1}{4}\gamma_i + \frac{1}{4}\sum_{j=1}^{i-1}\gamma_j \\ &< \frac{1}{4}\gamma_i + \frac{1}{4}\Delta + \frac{1}{4}\sum_{j=1}^{i-1}\gamma_j \\ &= \hat{\gamma}_i\frac{1}{4} + (\sum_{j=1}^{i-1}\gamma_j)\frac{1}{4}\end{aligned}$$

Also, all constraints $j > i+1$ continue to hold with equality:

$$\begin{aligned}b^2\sum_{k=i+2}^{j}\gamma_k + (\hat{\gamma}_{i+1}+\hat{\gamma}_i)b^2 + b^2\sum_{k=2}^{i-1}\gamma_k &= b^2\sum_{k=2}^{j}\gamma_k \\ &= \frac{1}{4}\sum_{k=1}^{j-1}\gamma_k \\ &= \frac{1}{4}\sum_{k=i+2}^{j-1}\gamma_k + \frac{1}{4}(\hat{\gamma}_{i+1}+\hat{\gamma}_i) + \frac{1}{4}\sum_{j=1}^{i-1}\gamma_j\end{aligned}$$

Note that this new strategy entails strictly less cost for the principal contradicting the assertion that the initial plan solved the optimization problem. Therefore this line of reasoning establishes that all the IC constraints must hold with equality in the optimal solution.

If all IC constraints hold with equality, then we have $\gamma_i = \prod_{j=i+1}^{N}\delta^*_{j+1}(1-\delta^*_i)$ in the optimal solution.

Given that we have $\gamma = \prod_{j=i+1}^{N}\delta^*_{j+1}(1-\delta^*_i)$, we now show that $q_i = q_i^*$. Note that we can move mass from any period i to any other period j because the IC constraints hold with inequality.

Suppose that there is a period i such that $q_i < q_i^*$. This implies by construction that 1) $q_i^* > 0$ and 2) $q_j^* = \bar{q}$ for all $j < i$. Therefore, $q_j \le q_j^*$ for all $j < i$. Also we know that $\sum_{i=1}^{N}(q_i\prod_{j=i+1}^{N}(1-q_j)) \ge p = \sum_{i=1}^{N}(q_i^*\prod_{j=i+1}^{N}(1-q_j^*))$. Hence, there must be a period $k > i$ such that $q_k > q_k^*$. However, then cost can be reduced by decreasing q_k and increasing q_i by $\epsilon > 0$ sufficiently small. □

Proof the the equilibrium is also agent optimality. Assume that the assessment σ^* is agent optimal for all reputation levels $p > 1-(1-\bar{q})^{i-1}$ in the i stage communication game. Under this induction hypothesis, we show that σ^* is agent optimal in the $i+1$ stage communication game for all reputation levels $p > 1-(1-\bar{q})^i$.

Fix an assessment σ. We will show that the cost under assessment σ^* is smaller than the cost under assessment σ given the induction hypothesis.

Suppose that the agent plays the biased action with probability zero in period $i+1$. The assumption on p implies that the agent must play the biased action with probability strictly greater than $\bar{q}$ in some period $j < i+1$. However, the unravelling argument presented in Lemma 4 (Step 1) implies that the agent will play the biased action with probability one also in period $i+1$ leading to a contradiction.

Suppose that the agent plays the biased action with total probability $q_{i+1} > \bar{q}$ in period $i+1$. This implies that the agent's cost is $1/4$ since the principal cannot communicate in period $i+1$ given that $q_{i+1} > \bar{q}$. However, the agent's cost under σ^* is at most $1/4$.

Suppose that the agent plays the biased action with total probability $q_{i+1} \leq \bar{q}$ in period $i+1$. In this case the agent's reputation following the unbiased action is $p_i = 1 - \frac{1-p}{1-q_{i+1}}$. Note that $q_{i+1} \leq \bar{q}$ implies that $p_i > 1-(1-\bar{q})^{i-1}$. There are two cases to consider: (i) If $q^*_{i+1} > \bar{q}$, then $q_{i+1} \leq \bar{q}$ implies that $q_j > \bar{q}$ for some $j < i+1$. However, then an unravelling argument (see Lemma 4 Step 1) implies that $q_{i+1} = p > \bar{q}$ leading to a contradiction. (ii) If $q^*_{i+1} \leq \bar{q}$, then the cost under σ^* is equal to $\delta^*_{i+1}/4$. The cost under σ is $\delta_{i+1}/4$ because the agent must play the biased action with positive probability in period $i+1$. (Otherwise $p > 1-(1-\bar{q})^i$ implies that $q_j > \bar{q}$ for some $j < i+1$. However, then the unravelling argument shows that $q_{i+1} = p > \bar{q}$ leading to a contradiction.) The fact that the agent is indifferent between the biased and the unbiased action implies the following equalities:

$$\begin{aligned} \frac{1}{4}\delta &= (1-\delta)b^2 + \delta C^A(i,\sigma) \\ \frac{1}{4}\delta^* &= (1-\delta^*)b^2 + \delta^* C^A(i,\sigma^*) \end{aligned}$$

where $C^A(i,\sigma)$ is the agent's cost in the continuation game under the strategy σ. The fact that $C^A(i,\sigma^*) \leq C^A(i,\sigma)$ implies that

$$\delta\frac{1}{4} \geq (1-\delta)b^2 + \delta C^A(i,\sigma^*).$$

Therefore

$$(\delta-\delta^*)(\frac{1}{4} + b^2 - C^A(i,\sigma^*)) \geq 0$$

However, because $C^A(i,\sigma^*) < \frac{1}{4} + b^2$ we have $\delta \geq \delta^*$ showing that the cost under σ exceeds the cost under σ^*.

We now complete the inductive argument by showing that σ^* is agent optimal for $i = 2$ if $p > 1-(1-\bar{q})^{2-1} = \bar{q}$. This conclusion follows immediately from the argument above because for any two strategy profiles σ and σ^*we have $C^A(1,\sigma) = C^A(1,\sigma^*) = 1/4$ if the reputation $p_1 > \bar{q}$ in period $i = 1$ and $C^A(1,\sigma) \geq C^A(1,\sigma^*) = 0$, otherwise. □

This concludes the proof of Theorem 1. □

Proof. [Proof of Proposition 1] Define $D_1 = 1$, let $a = 4b^2$ and note that δ^*_i, $i = 2,\ldots,N$, is defined by the following system of equations:

$$\delta^*_i = \frac{a}{1+aD_{i-1}} \tag{7.1}$$

$$D_i = \delta^*_i D_{i-1} \tag{7.2}$$

for all $i = 2,\ldots,N$. This, in turn, can be reduced to the following difference equation with initial condition $D_1 = 1$:

$$D_i = \frac{aD_{i-1}}{1+aD_{i-1}},\ i = 2,\ldots,N. \tag{7.3}$$

Claim 1. *Unique solution to the difference equation given in (7.3) is given by*

$$D_i = \frac{a^{i-1}}{\sum_{j=0}^{i-1} a^j} \tag{7.4}$$

Proof. [Proof of Claim 1] Proof is by induction. $D_2 = a/(1+a)$, so it is true for $i = 2$. Suppose now that it is true for $2 \leq k \leq N-1$. Then

$$D_{k+1} = \frac{aD_k}{1+aD_k} = \frac{a\frac{a^{k-1}}{\sum_{j=0}^{k-1} a^j}}{1+a\frac{a^{k-1}}{\sum_{j=0}^{k-1} a^j}} = \frac{a^k}{\sum_{j=0}^{k} a^j}$$

which establishes the claim. □

Substituting (7.4) into (7.1), we obtain

$$\delta_i^* = \frac{a}{1+a\frac{a^{i-2}}{\sum_{j=0}^{i-2} a^j}} = \frac{\sum_{j=1}^{i-1} a^j}{\sum_{j=0}^{i-1} a^j}. \tag{7.5}$$

Claim 2.

$$\gamma_{N-i} = \frac{a^i}{\sum_{j=0}^{N-1} a^j}, \; i = 0,1,\dots,N-1. \tag{7.6}$$

Proof. [Proof of Claim 2] First, note that

$$\gamma_N = 1-\delta_N^* = 1 - \frac{\sum_{j=1}^{N-1} a^j}{\sum_{j=0}^{N-1} a^j} = \frac{1}{\sum_{j=0}^{N-1} a^j}$$

Second, by definition $\gamma_{N-i} = \delta_N^* \delta_{N-1}^* \dots \delta_{N-i+1}^* (1-\delta_{N-i}^*)$ for any $i = 1,\dots,N-1$. Again by definition $D_i = \delta_i^* \delta_{i-1}^* \dots \delta_2^*$, which implies that

$$\delta_N^* \delta_{N-1}^* \dots \delta_{N-i+1}^* = \frac{\delta_N^* \delta_{N-1}^* \dots \delta_2^*}{\delta_{N-i}^* \delta_{N-i-1}^* \dots \delta_2^*} = \frac{D_N}{D_{N-i}}$$

Therefore,

$$\begin{aligned} \gamma_{N-i} &= \frac{D_N}{D_{N-i}}(1-\delta_{N-i}^*) \\ &= \frac{\frac{a^{N-1}}{\sum_{j=0}^{N-1} a^j}}{\frac{a^{N-i-1}}{\sum_{j=0}^{N-i-1} a^j}} \left(1 - \frac{\sum_{j=1}^{N-i-1} a^j}{\sum_{j=0}^{N-i-1} a^j}\right) \\ &= \frac{a^i}{\sum_{j=0}^{N-1} a^j} \end{aligned}$$

for any $i = 1,\dots,N-1$. This proves the claim. □

It is now easy to show that growth rate of the importance parameter γ is $\ln a$ and that γ_N decreases in a. □

Proof. [Proof of Proposition 2] Let the prior be $p > \overline{q}$ and k the largest integer such that $p > 1-(1-\bar{q})^{k-1}$. Assume that $k \geq 2$. Theorem 1 and the discussion that follows it implies that, if $N \geq k$, the total cost is equal to

$$TC = (\gamma_1 + \cdots + \gamma_{k-1})\bar{q}b^2 + \gamma_k q_k b^2 < \bar{q}b^2$$

since $q_k \leq \bar{q}$. Total cost when $N < k$, on the other hand, is at least $\bar{q}b^2$, because the total probability of the biased action is greater than or equal to $\bar{q}$ in period N. This implies that it is strictly better to choose $N \geq k$ rather than $N < k$. Let $W_i = \gamma_1 + \cdots + \gamma_i$ and note that $W_k = D_N/D_k$ by definition. Since $\gamma_k = W_k - W_{k-1}$, total cost can be written as

$$\begin{aligned} TC &= [W_{k-1}(\bar{q} - q_k) + W_k q_k]b^2 \\ &= [\delta_k^*(\bar{q} - q_k) + q_k]\frac{D_N}{D_k}b^2. \end{aligned}$$

If $k = 1$ or $p \leq \overline{q}$, then the total cost is equal to $\gamma_1 pb^2 = D_N pb^2$. Equation (7.4) implies that D_N is strictly decreasing in N, which implies that the total cost is decreasing N. Furthermore, if $b > 1/2$, then $\lim_{N\to\infty} D_N = 1 - 1/a > 0$, which implies that the lower bound on the total cost is strictly positive. If $b < 1/2$, then $\lim_{N\to\infty} D_N = 0$. □

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