Market Selection and the Information Content of Prices

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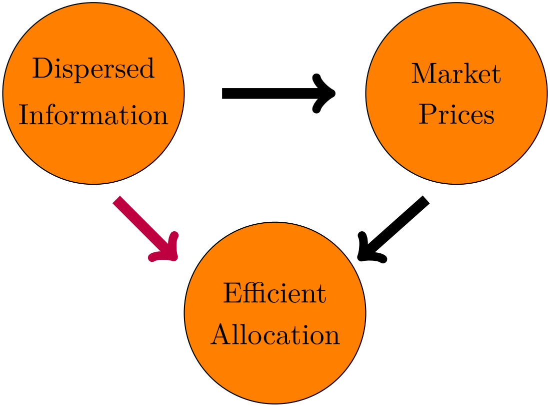
Motivation



Which markets better aggregate disperse information?
How should we design markets for informational efficiency?

Large Markets and Auctions

Rational expectations equilibria

- Existence of fully revealing RE equilibria
- Hayek, Radner, Grossman-Stiglitz

Auctions as a price formation process

- Uniform price common-value auction, many objects & bidders
- Information aggregated if goods trade in a single large auction market: Pesendorfer and Swinkels ('97, '00)

This paper: Bidders Self-Select across Multiple Markets

Market Selection and Frictions

Market Selection

- Multiple markets for real estate, skilled-unskilled labor markets, fragmented trading in stocks, commodities, bonds (primary vs secondary markets)
- Different markets have different institutional structures and frictions.
- Preparing and trading in a market might be costly.

Value of Alternative Option

- Exogenous outside option
- Endogenous outside option: value of best alternative market

Recent Interest in Information Aggregation

Information aggregation in markets, elections, betting markets

- Lauermann and Wolinsky (2014): Common-value auction with bidder solicitation: single good, state dependent number of bidders
- Valimaki and Murto (2015): Common-value auction with participation costs
- Ekmekci and Lauermann (2015): Election manipulation with endogenous participation costs
- Burguet and Sakovics (1999) and Pai (2010): Independent private values, competition among two auctioneers
- Atakan and Ekmekci (2014): Information aggregation can fail if there are ex-post actions/investments

Model Description

- Bidders choose between auction market s and an outside option r
  - n bidders, $n\kappa_s$ identical objects in market s, $\kappa_s < 1$
- Bidders' value $V = v \in \mathcal{V}$, $\Pr(V = v) = 1/|\mathcal{V}|$
  - Throughout the talk $\mathcal{V} = \{0, 1\}$
- Each bidder has unit demand.
  - Payoff equal to $V - P_s$ if buys an object in auction s
  - Zero if fails to buy an object in auction s
  - $u(r|V)$ if chooses r
  - $u(r|V = v)$ nondecreasing in v

Model Cont.

- Bidders receive private signals $\theta_i \in [0, 1]$
  - $\theta_i \sim F(\cdot|v)$ with density $f(\cdot|v)$, IID conditional on V
  - MLRP: For $v > v'$, $l(\theta) = f(\theta|v)/f(\theta|v')$ continuous and increasing in θ
  - Bounded information: $0 < \nu < l(\theta) < \frac{1}{\nu}$ for all θ
- After observing their signal, choose between s and r
- Auction s: closed bid, uniform $k_m + 1$st price auction
  - Bidders do not observe others' choice
  - If there are fewer bidders than the goods, then $P_s = 0$

Overview of Results

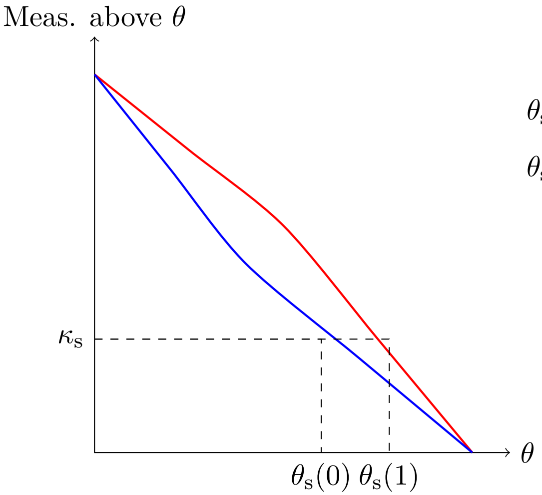
- Exogenous Outside Options: Suppose that the outside option is valuable for some types. In a large finite market
  - If κ_s is larger than a cutoff κ^*, then information is not aggregated in any equilibrium
  - In particular, if $\mathbb{E}[u(r|V)] = 0$ and $\mathrm{Var}[u(r|V)] > 0$ then information is not aggregated in any equilibrium
  - Equilibrium characterization that elaborates on the mechanism that leads to non aggregation
- Endogenous outside options: The outside option payoff profile can be generated by a concurrent auction market r with a reserve price

Market Selection and Equilibrium Bidding

- Market selection strategy, $a_s : [0,1] \to \{s, r\}$
- Bidding strategy, $b_s : [0,1] \to [0,1]$
- $\theta_s(v)$: Pivotal type in state v
- Equilibrium bidding function is nondecreasing
  - If θ prefers b' to $b < b'$, then $\theta' > \theta$ also prefers b' to $b < b'$
  - Conditional IID: Information about V obtained from the other $n-1$ signals, i.e., from $Y_s^{n-1}(k)$, is identical
  - MLRP: Type θ' more optimistic because $\theta' > \theta$

No Outside Option: PS '97

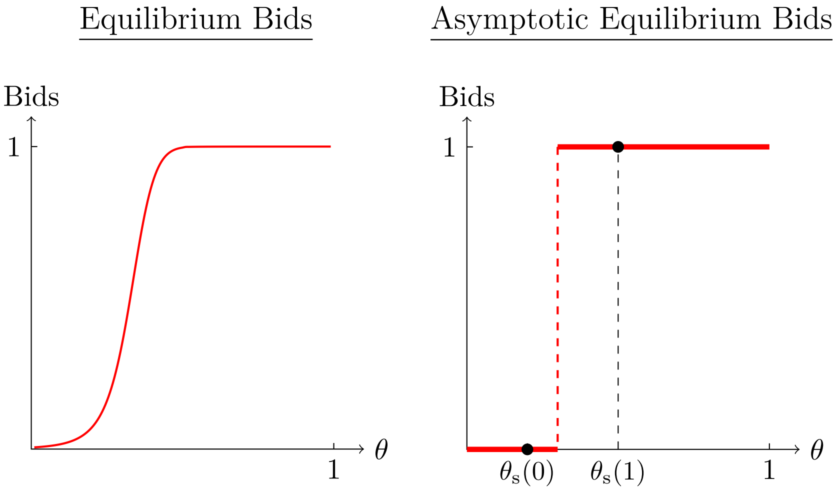
- $\mathbb{E}[\mathrm{u}(\mathrm{r}|\mathrm{V})] = 0$, $\mathrm{Var}[\mathrm{u}(\mathrm{r}|\mathrm{V})] = 0 \Rightarrow \mathrm{b}(\theta)$ increasing $\Rightarrow \mathrm{b}(\theta) =$ value conditional on pivotality



$\theta_s(1)$ pivotal if $V = 1$

$\theta_s(0)$ pivotal if $V = 0$

Information Aggregation: $P_s \xrightarrow{p} V$



Example: Non Aggregation

Suppose that $\mathbb{E}[\mathrm{u}(\mathrm{r}|\mathrm{V})] = 0$ but $\mathrm{Var}[\mathrm{u}(\mathrm{r}|\mathrm{V})] > 0$

$$\mathbb{E}[\mathrm{u}(\mathrm{r}|\mathrm{V})|\theta_{\mathrm{i}} = \theta] = \frac{\mathrm{l}(\theta)}{1 + \mathrm{l}(\theta)}\mathrm{u}(\mathrm{r}|\mathrm{V} = 1) + \frac{1}{1 + \mathrm{l}(\theta)}\mathrm{u}(\mathrm{r}|\mathrm{V} = 0)$$

Outside Option Valuable for Some θ

- If $\mathrm{l}(\theta) = 1$, then $\mathbb{E}[\mathrm{u}(\mathrm{r}|\mathrm{V})] = \mathbb{E}[\mathrm{u}(\mathrm{r}|\mathrm{V})|\theta_{\mathrm{i}} = \theta]$
  - Because $\frac{\mathrm{l}(\theta)}{1+\mathrm{l}(\theta)} = \frac{1}{2}$
- If $\mathrm{l}(\theta) > 1$, then $\mathbb{E}[\mathrm{u}(\mathrm{r}|\mathrm{V})|\theta_{\mathrm{i}} = \theta] > \mathbb{E}[\mathrm{u}(\mathrm{r}|\mathrm{V})] = 0$
  - Because $\frac{\mathrm{l}(\theta)}{1+\mathrm{l}(\theta)} > \frac{1}{2}$

Example: Suppose Information Aggregated

Market s not Valuable for any Type

- Information aggregation $\Rightarrow \lim \mathbb{E}[\mathrm{P}_s^n | \mathrm{V} = 1] = 1$
  $\lim \mathbb{E}[\mathrm{P}_s^n | \mathrm{V} = 0] = 0$
- Hence expected payoff (and variance) is equal to zero in market s.

Selection Effect

- Types θ such that $\mathrm{l}(\theta) > 1$ choose r
- Types θ such that $\mathrm{l}(\theta) < 1$ choose s

Example: Self-Selection ⇒ Non Aggregation

- Let θ^* be s.t. $\mathrm{l}(\theta^*) = 1$. Only $\theta < \theta^*$ choose s

$$\int_{\theta_{\mathrm{s}}^{\mathrm{n}}(1)}^{\theta^*} \mathrm{f}(\theta|1)\mathrm{d}\theta = \kappa_{\mathrm{s}} \text{ and } \int_{\theta_{\mathrm{s}}^{\mathrm{n}}(0)}^{\theta^*} \mathrm{f}(\theta|0)\mathrm{d}\theta = \kappa_{\mathrm{s}}$$

- Because $\mathrm{f}(\theta|1) < \mathrm{f}(\theta|0)$ for all $\theta < \theta^*$

$$\theta_{\mathrm{s}}^{\mathrm{n}}(1) < \theta_{\mathrm{s}}^{\mathrm{n}}(0)$$

- Information aggregation and LLN

$$\mathrm{b}_{\mathrm{s}}^{\mathrm{n}}(\theta_{\mathrm{s}}^{\mathrm{n}}(1)) \to 1 \text{ and } \mathrm{b}_{\mathrm{s}}^{\mathrm{n}}(\theta_{\mathrm{s}}^{\mathrm{n}}(0)) \to 0$$

- Monotonicity of Bidding

$$\mathrm{b}_{\mathrm{s}}^{\mathrm{n}}(\theta_{\mathrm{s}}^{\mathrm{n}}(1)) \leq \mathrm{b}_{\mathrm{s}}^{\mathrm{n}}(\theta_{\mathrm{s}}^{\mathrm{n}}(0))$$

a contradiction!

Information Non Aggregation

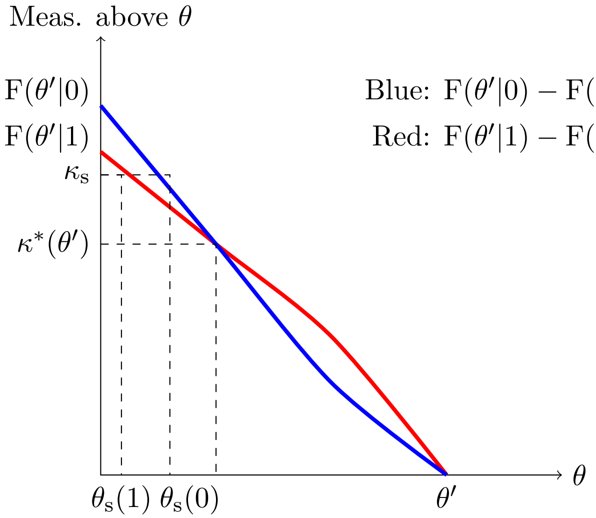
Theorem

Suppose that $\mathbb{E}[\mathrm{u}(\mathrm{r}|\mathrm{V})|\theta_{\mathrm{i}} = 1] > 0$ and let

$$\theta' = \inf\{\theta : \mathbb{E}[\mathrm{u}(\mathrm{r}|\mathrm{V})|\theta_{\mathrm{i}} = \theta] > 0\}.$$

1. There exists $\kappa^*(\theta') < 1$ such that if $\kappa_{\mathrm{s}} > \kappa^*(\theta')$, then no equilibrium sequence H aggregates information.
2. There is a cutoff $\bar{\mathrm{u}} > 0$ such that if $\mathrm{Var}[\mathrm{u}(\mathrm{r}|\mathrm{V})] < \bar{\mathrm{u}}$ and if $\kappa_{\mathrm{s}} < \kappa^*(\theta')$, then every equilibrium sequence H aggregates information.

Selection Reverses Ordering of Pivotal types



- If $\kappa_s > \kappa^*(\theta')$, then $F(\theta'|0) - F(\theta|0) > F(\theta'|1) - F(\theta|1)$

Sketch of Argument

- Suppose info is aggregated in s :
  - Price converges to the value
  - Profits disappear in both states
  - Types $\theta > \theta'$ select option r
- If κ_{s} is large, pivotal type in state 0 is larger than pivotal type in state 1
- Monotonicity of equilibrium bids and the ordering of pivotal types contradict info aggregation in market s

Structure of Equilibria

- $u(r|V=1) > 0$ and $u(r|V=0) \geq 0$. Unique equilibrium:
  - Information aggregation fails because of lack of competition
  - Number of bidders less than k with probability 1 and with positive probability if $V = 1$
  - Price $P_s = 0$ if $V = 0$ and has two point support $\{0, 1\}$ if $V = 1$
- $u(r|V=0) < 0 < u(r|V=1)$ and $Var[u(r|V)] < \bar{u}$
  - Sufficient competition but information aggregation fails
  - Pivotal types are arbitrarily close
  - The same set of types determine the price
  - Price distribution's support is continuous and identical in both states

Price Distribution

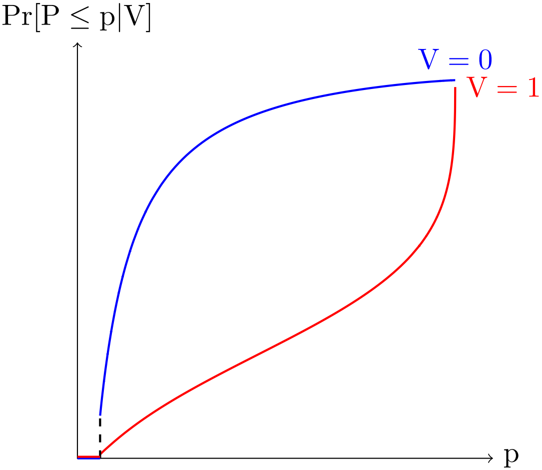


Figure: Limit Cumulative Price Distribution of V = 0 and V = 1.

Outside Option Valuable for a Subset of Types

Theorem

Suppose that $\mathbb{E}[\mathrm{u}(\mathrm{r}|\mathrm{V})|\theta_{\mathrm{i}} = 0] < 0$ and that $\kappa > \kappa^*(\theta')$ where θ' is the type defined in Theorem. There is a cutoff $\bar{\mathrm{u}} > 0$ such that if $\mathrm{Var}[\mathrm{u}(\mathrm{r}|\mathrm{V})] < \bar{\mathrm{u}}$, then for every equilibrium sequence

- $\mathrm{F_s}(1|\mathrm{v}) > \kappa$ for $\mathrm{v} = 0, 1$
- The pivotal types are arbitrarily close
- There is θ^{n} arbitrarily close to the pivotal types such that $\mathrm{b}^{\mathrm{n}}_{\mathrm{s}}(\theta)$ is increasing at all $\theta > \theta^{\mathrm{n}}$

Arbitrarily Close Pivotal Types

- Which type clears the market?

$$\sqrt{n}(\bar{F}_s^n(Y_s^n(k)|v) - \kappa_s) \xrightarrow{d} N(0, \kappa_s(1-\kappa_s))$$

- Pivotal types are arbitrarily close:

$$\lim \sqrt{n}|\bar{F}_s^n(\theta_s^n(1)|1) - \bar{F}_s^n(\theta_s^n(0)|1)| = \lim \sqrt{n}|\kappa_s - \bar{F}_s^n(\theta_s^n(0)|1)| < \infty$$

- If the auction price clears at the bid of some θ^n with positive probability in state $V = 1$, then it clears at this types bid also in state $V = 0$

Asymptotics of Pivotal Types

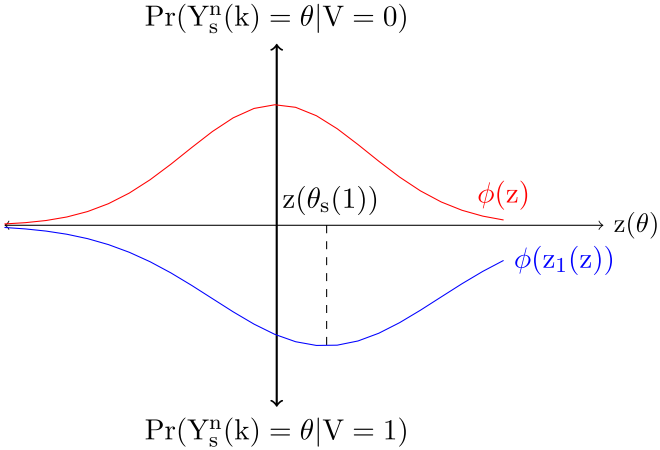


Figure: The x-axis measures the normalized distance between θ and $\theta_s(0)$. Also, $z_1(z)$, which is a affine function, is the normalized distance between θ and $\theta_s(1)$.

Sketch of Argument

- Suppose that $\lim \sqrt{n}|\bar{F}_s^n(\theta_s^n(1)|v) - \bar{F}_s^n(\theta_s^n(0)|v)| \to \infty$
- If $\theta_s^n(1)$ and $\theta_s^n(0)$ submit distinct bids, then
  - $b^n(\theta_s^n(1)) \to 1$ and $b^n(\theta_s^n(0)) \to 0$
  - Hence, information is aggregated
  - Therefore the pivotal types must submit the same "pooling" bid
- The argument that follows shows that pooling by the pivotal types is not possible

Argument Continued

- If $\theta_s^n(1)$ and $\theta_s^n(0)$ pool, then the pooling bid is large and the auction clears at the pooling bid by LLN.
  - In particular, $b_p x > \bar{u} > -u(r|V = 0)$ where x is the probability of winning at pooling
  - E.g., if $b_p < -u(r|0)$, then all bidders would prefer bidding just above the pooling bid and winning an object at the pooling bid to the outside option
- Single crossing:
  - Suppose $x\mathbb{E}[P_s|V = 0] > -u(r|V = 0)$ and some θ picks s over r, then all $\theta' > \theta$ also picks s over r
- All $\theta > \theta_s(0)$ choosing s precludes atoms and implies that information is aggregated in market s

Equilibrium Construction

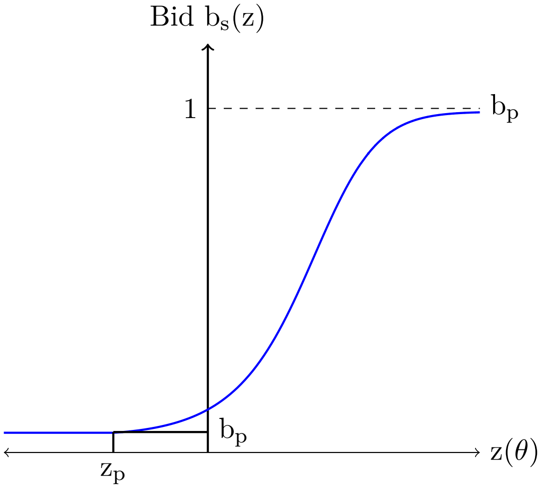


Figure: Bidding: $\beta(z) = \lim \beta^n(\theta^n)$ where $z = \lim \frac{\sqrt{n}(\kappa_s - \bar{F}(\theta^n|0))}{\sqrt{\kappa_s(1-\kappa_s)}}$.

Outside Option Valuable for All Types

Theorem

If $\mathrm{u}(\mathrm{r}|0) > 0$, then in the unique equilibrium:

1. There is a certain cutoff type $\hat{\theta}^{\mathrm{n}}$ such that all types $\theta < \hat{\theta}^{\mathrm{n}}$ opt for the outside option.
2. Bidding function $\beta_{\mathrm{s}}^{\mathrm{n}}(\theta)$ is strictly increasing
3. All bids converge to one, i.e., $\beta_{\mathrm{s}}^{\mathrm{n}}(\theta) \to 1 \; \forall \theta > \hat{\theta}$
4. $\lim |\mathrm{F}_{\mathrm{s}}^{\mathrm{n}}(1|1) - \kappa_{\mathrm{s}}|\sqrt{\mathrm{n}} < \infty$ and $\mathrm{F}_{\mathrm{s}}(1|0) < \kappa_{\mathrm{s}}$.
5. If $\mathrm{V} = 0$, the price converges to zero. If $\mathrm{V} = 1$, then the price converges to a random variable which is equal to zero with probability $\mathrm{q} > 0$ and equal to one with the remaining probability.

Sketch of Argument

- Pessimists choose r and optimists choose s because s is the riskier option
- Types who bid in the auction bid aggressively: $\beta_s^n(\theta) \to 1$ for all types θ above the pivotal type in state 1
- If $(F_s^n(1|1) - \kappa_s)\sqrt{n} \to +\infty$, then $P_s \to 1$ in state 1. But then nobody would choose the auction
- If $(F_s^n(1|1) - \kappa_s)\sqrt{n} \to -\infty$, then $P_s \to 0$ in both states. But then everybody would choose the auction
- Only alternative $F_s^n(1|1) - \kappa_s$ on the order of $1/\sqrt{n}$

Endogenous Outside Options

- What kind of markets can generate the outside option payoff profile under consideration?
- Suppose that r is also an auction. Then,
  - An identical auction to s with a reserve price generate an outside option payoff profile similar to the one under consideration. We study such a model next
- The following also generate similar payoff profiles:
  - An all-pay auction
  - A pay your bid auction (first price auction)

Endogenous Outside Option

- Bidders decide between auction market s and auction market r
- Additional $n\kappa_r$ identical objects in market r, $\kappa_s + \kappa_r < 1$
- Market r identical to market s except for a reserve price $c \geq 0$
- Let

$$\theta^* = \inf\{\theta : \mathbb{E}[V - c|\theta_i = \theta] > 0\}.$$

Endogenous Outside Option

Theorem

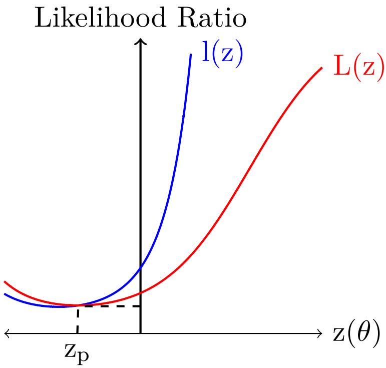
Assume that $\kappa_s > \kappa^*(\theta^*)$ and that $c < \bar{c}$.

1. In market r, $\lim \sqrt{n}|F_r^n(1|1) - \kappa_r| < \infty$ and $F_r(1|0) < \kappa_r$.
2. If $V = 0$, then the price in market r converges to c. If $V = 1$, then the price in market r converges to a random variable which is equal to zero with probability $q > 0$ and equal to one with the remaining probability.
3. The expected prices are equal across states and markets. In particular, $\lim \mathbb{E}[P_s^n|V = 0] = \lim \mathbb{E}[P_r^n|V = 0] = c$ and $\lim \mathbb{E}[P_r^n|V = 1] = \lim \mathbb{E}[P_s^n|V = 1]$
4. In market s, $\lim \sqrt{n}|F_s(\theta_s^n(1)|1) - F_s(\theta_s^n(0)|1)| < \infty$.

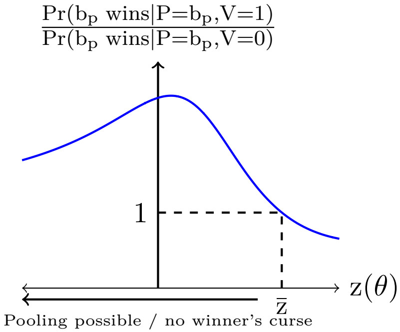
Conclusion

- Informational contagion and spillovers. Mechanism of propagation:
  - Variance in the outside option or frictional market (almost) by assumption.
  - Markets serve as state dependent outside options for each other.
  - Information failure propagates as pessimistic bidders select the safer market.
- Pesendorfer and Swinkels (97): Information not socially valuable. Reasons for uninformative prices:
  - Grossman and Stiglitz (1980): Costly information.
  - Atakan and Ekmekci (2014): Information valuable because of ex-post actions.
  - This paper: If information is valuable because of uncertain gains from trade.
  - In all these cases information is either costly or socially valuable.

Equilibrium Construction



(a) Likelihood Functions and the determination of θ_p^n:
$l(z) = \lim l(Y_s^{n-1}(k) = \theta^n, \theta_i = \theta^n)$,
$L(z) = \lim l(Y_s^{n-1}(k) \leq \theta^n, \theta_i = \theta^n)$



(b) Loser's and Winner's Curse at the Pooling bid.