Information Aggregation in Auctions with Costly Information*

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Abstract

We study a common-value auction in which a large number of identical, indivisible objects are sold to a large number of ex-ante identical bidders with unit demand. Bidders are initially uninformed but can acquire information from multiple sources that differ in accuracy and cost. We define a cost-accuracy ratio for each available source of information. The minimum value of this cost-accuracy ratio among all information sources fully determines the limit price distribution and the information content of the auction's price. Information is aggregated if and only if the minimum cost-accuracy ratio is equal to zero. We also characterize all equilibria of the auction for posterior separable information costs with a sufficiently rich set of experiments. In this case, information is aggregated if and only if the cost function is differentiable at the prior.

Keywords: Auctions, Large markets, Information Aggregation, Cost of Information.

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1. Introduction

Trust in markets is partially predicated on the belief that competitive prices summarize all available information (Fama (1970)). However, theoretical support for this belief is equivocal. On the one hand, past work shows that the price in a competitive market or a large common-value auction fully reveals an unknown state of the world if sufficient information is dispersed among bidders.[1] On the other hand, past work also shows that markets that aggregate dispersed information provide no incentive for agents to invest in information (see Grossman and Stiglitz (1980), Matthews (1984), Pesendorfer and Swinkels (2000), and Jackson (2003)). Therefore, the informativeness of market prices depends on how the tension between information acquisition and information aggregation resolves. In this paper, we study the equilibrium resolution of this tension in a common-value auction where bidders can acquire costly information from multiple sources. We provide answers to the following questions: What is the accuracy of the information chosen by each bidder? How much information is acquired in aggregate? How much of this information is reflected in the auction price, that is, does the auction price aggregate information? How does the information content of the auction price change as the set of information sources and the cost of acquiring information vary?

In the model that we analyze, n players, each with unit demand, simultaneously bid on k identical, indivisible objects that are on sale through a uniform-price auction. The unknown value of the object $V \in \{v_0, v_1\}$ (or the state) is the same for all of the bidders. This value is either equal to zero or one and both outcomes are equally likely. Before participating in the auction, each bidder can acquire information about the object on sale by choosing an information source (experiment) F from the set of available information sources at a cost equal to $C(F) > 0$. Following the literature on information design, we model each experiment as a distribution over posteriors that satisfies Bayesian plausibility (see Kamenica and Gentzkow (2009)). More precisely, a bidder who chooses experiment F draws a type $\theta \in [0, 1]$ from the distribution function F and this type is her posterior belief that the object's value is equal to one.

We begin our analysis by focusing on the accuracy of the information acquired by each bidder in a symmetric equilibrium. We define the accuracy of an experiment, $A(F)$, as the probability of correct optimism minus the probability of incorrect optimism, or more precisely, $A(F)$ is given by the difference between the

[1]See Grossman and Stiglitz (1976), Wilson (1977), Radner (1979), Milgrom (1979), and Pesendorfer and Swinkels (1997).

probability that a bidder draws a posterior that exceeds her prior in state v_1, i.e., $1 - F(1/2|v_1)$, and the same probability in state v_0. In a key result, we show that the incentive to acquire accurate information is determined by the extent to which the price diverges from value. More specifically, the payoff from choosing any experiment F converges to

$$\text{Accuracy of experiment } F \times \text{Divergence of price from value},$$

where the divergence of price from value is given by $\sum_{i\in\{0,1\}} |v_i - \mathbb{E}[P|v_i]|/4$ and $\mathbb{E}[P|v_i]$ denotes the expected price in state v_i. Hence, the value of information is asymptotically linear in its accuracy, and the marginal value of accuracy is equal to the divergence of price from value. In equilibrium, each bidder chooses an information source with strictly positive accuracy unless the marginal cost of accuracy is prohibitively high. However, the accuracy of the experiment chosen converges to zero as the market grows large. This is because otherwise, the auction price would converge to value (see Pesendorfer and Swinkels (1997) or Atakan and Ekmekci (2021)), leaving no incentive to acquire information. In equilibrium, each bidder chooses the accuracy of her information optimally by equating the marginal value of accuracy (i.e., divergence of price from value) with the marginal cost of accuracy. Therefore, the divergence of price from value is determined by the marginal cost of accuracy evaluated at zero as the auction grows large. In fact, we show that the marginal cost of accuracy at zero is equal to the infimum of $C(F)/A(F)$ taken over the set of all information sources (the cost-accuracy ratio). Hence, the divergence of price from value converges to the cost-accuracy ratio as the auction grows large.

In our main result (Theorem 1), we turn to whether the auction price aggregates information or more precisely, whether an outsider can perfectly identify the unknown state by only observing the auction price as the auction grows large. We show that bidders acquire a sufficient amount of information in aggregate to identify the state and the auction price perfectly aggregates this information if the cost-accuracy ratio is equal to zero. The argument for this result relies on our finding that the divergence of price from value converges to the cost-accuracy ratio. Information is aggregated because the divergence of price from value converges to zero if the cost-accuracy ratio is equal to zero. If, on the other hand, the cost-accuracy ratio is greater than zero, then the auction price does not aggregate information. For this case, we use the central limit theorem to derive the limit price distribution, and we show that the cost-accuracy ratio uniquely determines the limit price distribution. We use the price distribution to argue that 1) An outside observer could identify the state if she observed the realized bid distribution, in other words, bidders acquire a sufficient amount of information to identify the state, 2) The equilibrium price does not reveal the state but is nevertheless an informative signal about the state, 3) The informativeness of the auction price is (Blackwell (1953)) decreasing in the cost-accuracy ratio.

Our equilibrium characterization provides further insight into how information aggregates and how it fails. Theorem 1 implies that information aggregation fails if there are only finitely many information sources.[2] In this case, bidders acquire information from the source with the smallest cost-accuracy ratio with probability $\mu > 0$ and remain uninformed with the remaining probability for all sufficiently large n. As the market grows large, the accuracy of the information that any informed bidder has remains constant, the fraction of bidders who are informed converges zero, but the expected number of informed bidders converges to infinity. The equilibrium amount of noise generated by uninformed bids hinders information aggregation just enough to incentivize bidders to acquire information from the source with the smallest cost-accuracy ratio.

Information aggregation can also fail with infinitely many information sources. We focus on this case by assuming that the set of experiments includes all distribution functions over some non-empty interval. To provide a precise description of equilibrium behavior, we further assume that the cost of information is posterior-separable, that is, $C(F) = \int c(\theta) DF(\theta)$ for some convex function c.[3] Under these assumptions, we show that the price aggregates information if and only if the function c is differentiable at the prior ($\theta = 1/2$) and we characterize the equilibrium type distribution through a differential equation. Our characterization shows that the support of the equilibrium type distribution is an interval (i.e., the set of types is infinite) and the type distribution is atomless at θ if and only if $c(\theta)$ is not differentiable at θ. Hence, the type distribution features an atom at the prior if and only if information aggregation fails. Several qualitative insights follow from these findings: All bidders acquire accurate information, but the accuracy of this information converges to zero along equilibrium sequences that aggregate information. In contrast, a vanishing fraction of bidders acquire relatively accurate information while all others remain uninformed along equilibrium sequences that fail to aggregate information. As is the case with finitely many information sources, the

[2] This is because the cost-accuracy ratio is positive with finitely many sources.

[3] There is a large literature that studies how to model the cost of information and in particular posterior separable cost of information. See, for example, Sims (2003; 2010), Caplin et al. (2022), Denti (2022), and Pomatto et al. (2020).

equilibrium amount of noise generated by uninformed bids incentivizes bidders to acquire information.

1.1. Related literature. Our paper is closely related to past work that studies information aggregation in common-value auctions. Prominently, Wilson (1977), Milgrom (1979), and Pesendorfer and Swinkels (1997) study auctions where all bidders draw costless signals from the same distribution. Wilson (1977)and Milgrom (1979) show that the price aggregates information as the number of bidders grows large in an auction for a single object if there are arbitrarily precise signals. The paper most closely related to ours, Pesendorfer and Swinkels (1997), further shows that information is aggregated under minimal assumptions on the signal distribution if the number of objects and the number of losers in an auction both converge to infinity. However, past work also demonstrates that these results are sensitive to introducing information costs. In particular, Matthews (1984) and Pesendorfer and Swinkels (2000) argue, through examples, that information aggregation can fail with costly information for reasons similar to Grossman and Stiglitz (1980). Jackson (2003) further shows that this is a general phenomenon if agents have access to only a single costly source of information. In contrast to these papers, we allow agents to acquire costly information flexibly from a rich set of experiments. We show that information is aggregated under mild conditions on the cost of information (posterior separability and smoothness) if the set of experiments is sufficiently rich. Moreover, we provide a necessary and sufficient condition for information aggregation and quantify the information content of price when information is not aggregated.

Past work by Persico (2000), Bobkova (2021), and Kim and Koh (2022) also study flexible information acquisition in auctions. Kim and Koh (2022) characterizes the unique equilibrium in a first-price auction for a single object where bidders have independent private values. Persico (2000) and Bobkova (2021) compare the incentives to acquire information in first and second-price auctions for a single object. In contrast, our main focus is on information aggregation in uniform-price auctions, and our equilibrium characterization differs from Kim and Koh (2022) since we study an auction where multiple common-value objects are on sale.[4]

The paper is organized as follows: Section 2 formally introduces our model; Section 3.1 presents our necessary and sufficient condition for information aggre-

[4]Costly information acquisition has been studied in a other areas of economics also. See, for example, Bergemann and Valimaki (2002), Crémer et al. (2009), Shi (2012) and Bikhchandani and Obara (2017) in mechanism design, Martinelli (2006) and Gerardi and Yariv (2008) in voting, Crémer and Khalil (1992) and Szalay (2009) in principal-agent-settings, Ekmekci and Kos (2020) in signaling games, and Ravid et al. (2022) in bilateral trade.

gation and derives the limit price distribution when information is not aggregated; Section 3.2 characterizes equilibrium behavior when the cost of information is posterior separable, Section 3.3 generalizes some of the findings presented in Section 3.2 to more general cost functions, and Section 4 concludes.

2. The Model

We study a sequence of common-value auctions, $\{\Gamma^n\}_{\lfloor \kappa n \rfloor = 1}^{\infty}$, where each auction has n bidders and $\lfloor \kappa n \rfloor$ identical objects with $\kappa \in (0,1)$.[5] The object's value is a random variable V drawn from the set $\{v_0, v_1\}$. Each bidder has unit demand and puts value V on a single object and value 0 on any further objects. The value V (or the state) is common across players but is unknown. The utility of a bidder who wins a good at price P equals $V - P$. We set $v_0 = 0$ and $v_1 = 1$, for simplicity.

Bidders are uninformed about the state and share a common prior equal to 1/2. However, each bidder can purchase information pertaining to the state at a cost. Bidders have access to a family of experiments $\mathcal{F} \subset \Delta[0,1]$. Each experiment $F \in \mathcal{F}$ is a probability distribution over posterior beliefs $\theta = \Pr\{V = v_1\}$ that satisfies Bayesian plausibility, that is, $\int_{[0,1]} \theta dF(\theta) = 1/2$.[6] We denote the cost of experiment F by $C(F)$.

In the auction Γ^n, each bidder privately chooses an experiment $F \in \mathcal{F}$ and privately observes her type (posterior) $\theta \in \Theta$. All bidders simultaneously bid in the auction after observing their private types. The $\lfloor \kappa n \rfloor$ highest bidders receive an object and pay a uniform price P^n which equals the $\lfloor \kappa n \rfloor + 1$st highest bid. We denote by $Y^n(k+1)$ the random variable that represents the $k+1$st highest bid in the auction. Hence, the auction price $P^n = Y^n(\lfloor \kappa n \rfloor + 1)$.

2.1. Strategies, Payoffs, and Equilibrium.

An experimentation strategy for player i is a probability measure $\mu_i \in \Delta(\mathcal{F})$. Each experimentation strategy μ induces a distribution over types $F(\theta;\mu) := \int_{\mathcal{F}} F(\theta) d\mu$ at a cost equal to $C(\mu) := \int_{\mathcal{F}} C(F) d\mu$. The bidding behavior of player i is represented by a distributional strategy $H_i \in \Delta(\Theta \times [0,\infty))$. The distribution strategy H_i is a probability measure over type and bid pairs $(\theta, b) \in \Theta \times [0,\infty)$ (see Milgrom and Weber (1985)) that satisfies $H_i([0,\theta], [0,\infty)) = F(\theta;\mu_i)$.[7] We say that the bid distribution is monotone if $b \in \text{supp } H(\theta)$ implies that $b > b'$ for any $b' \in \text{supp } H_i(\theta')$

[5] The largest integer not greater than x is denoted by $\lfloor x \rfloor$.

[6] Note that in our formulation instead of working with a signal we work directly with the posteriors generated by a signal.

[7] Note that we do not allow bidder i to condition her bidding strategy on the realization of the experiment in the support of μ_i. This is without loss of generality because other bidders do not observe the experiment chosen by bidder i. Hence, allowing for H_i to depend on the realization would not change the set of equilibrium outcomes.

and $\theta' < \theta$. A symmetric strategy profile is one in which all players use the same experimentation strategy μ and the same bidding strategy H. We refer to a symmetric strategy profile by (μ, H). We focus on Nash equilibria of the game Γ where each player uses a symmetric strategy (μ, H). We call a subsequence of symmetric equilibria $\{\mu^j, H^j\}_{j=1}^{\infty}$ of the auctions $\{\Gamma^{n_j}\}_{j=1}^{\infty}$ an equilibrium sequence.

We represent by $\Pr^H$ the joint probability distribution induced by the symmetric strategy H over states of the world, signal and bid distributions, allocations, and prices. We denote the payoff from submitting a bid equal to b in state v_i, given that all others bidders follow symmetric strategy (μ, H), by $u^H(b, v_i)$. Similarly, the expected payoff from a mixed bidding strategy $\sigma \in \Delta([0,1])$ in state v_i is $u^H(\sigma, v_i) := \int u^H(b, v_i) d\sigma(b)$, and the expected payoff of type θ from bidding according to $\sigma \in \Delta([0,1])$ is $u^H(\sigma, \theta) := \theta u^H(\sigma, v_1) + (1-\theta) u^H(\sigma, v_0)$. To simplify notation, when (μ, H) is an equilibrium, we use u instead of u^H for the equilibrium payoff when suppressing the superscript leads to no ambiguity. Finally, we denote the expected equilibrium payoff for a bidder by $U(\mu, H) := \int_\Theta u^H(H(\theta), \theta) dF(\theta, \mu) - C(\mu)$.

2.2. Information Aggregation and the Information Content of Prices. Information is aggregated in the auction if the auction's equilibrium price conveys precise information about the state of the world to an outsider who only observes the price. Suppose that the random variable P^n describes the price in an auction where bidders behave according to strategy (μ^n, H^n). Note that, (i) if $\Pr(V = v_1 | P^n = p)$ is arbitrarily close to zero for large n, then an outsider who observes price p learns that the state is v_0. Similarly, (ii) if $\Pr(v_1 | P^n = p)$ is arbitrarily close to one, then an outsider who observes price p learns that the state is v_1. The auction aggregates information if the probability of observing a price satisfying either (i) or (ii) is arbitrarily close to one along any equilibrium sequence. Atakan and Ekmekci (2021) showed that information aggregates along any equilibrium sequence if and only if the equilibrium price P^n converges in probability to V. In light of this result, we use price converging to value and information aggregation interchangeably.

Definition 1 (Information Aggregation)**.** We say that information is aggregated if $P^n \to V$ in probability across every equilibrium sequence and information aggregation fails if there is no equilibrium sequence along which $P^n \to V$ in probability.

The distribution of an outsider's posterior upon observing the auction price, that is, the distribution of the random variable $\Pr(v_1 | P)$, quantifies the information content of the auction price. In cases where information is not aggregated, we use the following definition to rank price distributions in terms of informativeness.

Definition 2 (Blackwell Monotonicity)**.** For any collection of random variables (or prices) $P(x)$ indexed by a parameter $x \in [0, \bar{x}]$, we say that the Blackwell (1953) informativeness of the auction price is increasing in x if $x' > x$ implies that $\Pr(v_1|P(x))$ is a mean preserving spread of $\Pr(v_1|P(x'))$, that is, if the distribution over posteriors induced by observing $P(x)$ is a mean preserving spread of the distribution of posteriors induced by observing $P(x')$.

2.3. The Family of Experiments. Recall that any experiment $F \in \mathcal{F}$ is a distribution over posteriors that satisfies Bayesian plausibility. If $F \in \mathcal{F}$ has a density, then we denote it by f, and Bayes' rule implies that $f(\theta|v_1) = 2\theta f(\theta)$ and $f(\theta|v_0) = 2(1-\theta) f(\theta)$. Throughout the paper, we assume that all the experiments in $\mathcal{F}$ are independent conditional on the state, and if two bidders choose the same experiment F, then they draw types that are independent conditional on the state. Moreover, we make the following two assumptions on the cost of information: 1) An uninformative experiment F° that puts all of its mass on belief $\theta = 1/2$ is available for free, and 2) Information is costly, that is, if $F \neq F^\circ$, then $C(F) > 0$. The first assumption allows each bidder to bid in the auction at no cost. The second assumption is natural in our context since our goal is to understand whether the auction can incentivize bidders to acquire costly information while simultaneously aggregating this information. If this assumption is not satisfied, then the analysis of Pesendorfer and Swinkels (1997) or Theorem 1 (presented further below) implies that information is aggregated. To ensure that an equilibrium exists, we assume that the set of experiments $\mathcal{F}$ is compact and the information cost function is continuous in the weak* topology.

The cost-accuracy ratio for a particular experiment $F \in \mathcal{F} \setminus \{F^\circ\}$ is given by

$$CA_F := C(F)/A(F),$$

where $A(F) := \bar{F}(1/2|v_1) - \bar{F}(1/2|v_0)$ and $\bar{F}(1/2|v_i) := 1 - F(1/2|v_i)$.[8] We define the cost-accuracy ratio

$$CA := \inf_{\mathcal{F}\setminus\{F^\circ\}} CA_F$$

as the smallest such ratio.

[8] Note that this ratio is not defined for the uninformative experiment F° as both the numerator and the denominator is equal to zero.

In the next section, we will provide more intuition for why we focus on this particular notion of accuracy. Below we give an economic interpretation of the cost-accuracy ratio by showing that it is equal to the marginal cost of accuracy evaluated at zero. Define the cost of accuracy as

$$\hat{C}(a) := \min_{\mu \in \Delta(\mathcal{F})} \left\{ C(\mu) : \bar{F}(1/2|v_1;\mu) - \bar{F}(1/2|v_0;\mu) \geq a \right\}$$

and the marginal cost of accuracy at zero as $\hat{C}'(0) := \liminf_{a \to 0} \hat{C}(a)/a$. Note that $\hat{C}'(0) \geq CA$ because $\hat{C}(a)/a \geq CA$ for each $a > 0$. We now argue that $\hat{C}'(0) = CA$. There are two cases to consider: There is an experiment $F^* \in \mathcal{F} \setminus \{F^\circ\}$ such that $CA = CA_{F^*}$. In this case, for each $a < A(F^*)$ we have

$$\frac{\hat{C}(a)}{a} \geq \frac{C(F^*)}{A(F^*)} = \frac{\mu C(F^*) + (1-\mu) C(F^\circ)}{\mu A(F^*) + (1-\mu) A(F^\circ)}$$

where $\mu = a/A(F^*)$. Therefore, $\hat{C}'(0) = CA$. Alternatively, there is no such experiment. In this case, $CA = \liminf_{F \to F^\circ} CA_F$. However, $CA_F \geq \hat{C}(A(F))/A(F)$ for each experiment F. Therefore, $\liminf_{F \to F^\circ} CA_F = \hat{C}'(0)$.

3. Main Results

3.1. Information Content of Prices. In this subsection, we first show that the auction aggregates information if and only if the cost-accuracy ratio is equal to zero (Theorem 1). We then turn to the cases where information is not aggregated. For this case, we show that the cost-accuracy ratio fully determines the information content of the auction price and the limit price distribution (Theorem 2). Moreover, the information content of the auction price is Blackwell decreasing in the cost-accuracy ratio.

We begin with several lemmata that describe equilibrium behavior in the auction. The game that ensues after bidders acquire information is a standard common-value auction with a unique symmetric equilibrium (see Pesendorfer and Swinkels (1997)). In this equilibrium, all players submit a bid equal to 1/2 if the uninformative experiment is chosen, that is, if $F(\mu) = F^\circ$. On the other hand, if an informative experiment is chosen, then bidding is monotone, atomless, and involves each player bidding their expected value conditional on the event that they win an object at a price equal to their bid. The following lemma summarizes these findings, and further argues that bidders purchase information if the cost of information is not prohibitively high.

Lemma 1. *If $CA < 1/4$, then $F(\mu) \neq F^\circ$, the bid distribution H is monotone,* *atomless, and for any type θ and bid $b \in$ supp $H(\theta)$ we have*

$$b = \mathbb{E}[V|Y^{n-1}(\lfloor \kappa n \rfloor) = b, \theta_i = \theta)]$$

in any symmetric equilibrium (μ, H) of Γ^n.

Proof. We first argue that $F(\mu) \neq F^\circ$ when $CA < 1/4$. Assume to the contrary that $F(\mu^n) = F^\circ$. Then all players would bid $1/2$ and receive an expected payoff equal to zero in the unique symmetric equilibrium of the auction. The following strategy is a profitable deviation: Pick experiment F such that $CA_F < 1/4$, bid 1 if $\theta > 1/2$, and bid 0 if $\theta \leq 1/2$. This strategy delivers the following payoff:

$$\frac{\bar{F}(1/2|v_1)}{2} \underbrace{(1 - 1/2)}_{\text{Payoff in state } v_1} + \frac{\bar{F}(1/2|v_0)}{2} \underbrace{(0 - 1/2)}_{\text{Payoff in state } v_0} - C(F) = \frac{\bar{F}(1/2|v_1) - \bar{F}(1/2|v_0)}{4} - C(F) > 0$$

where the strict inequality follows from $CA_F = C(F)/\left(\bar{F}(1/2|v_1) - \bar{F}(1/2|v_0)\right) < 1/4$.

The equilibrium type distribution $F(\mu^n) \neq F_0$ satisfies weak MLRP. Therefore, the bid distribution is atomless, and bidding is monotone by Pesendorfer and Swinkels (1997), Lemma 6. Pesendorfer and Swinkels (1997), Theorem 1 further implies that any bid $b \in$ supp $H^n(\theta)$ satisfies $b = \mathbb{E}[V|Y^{n-1}(\lfloor \kappa n \rfloor) = b, \theta_i = \theta)]$. □

The previous lemma showed that bidders acquire information. Below we further argue that the accuracy of this information converges to zero as the auction grows arbitrarily large.

Lemma 2. *Along any equilibrium sequence $F(\mu^n)$ converges in distribution to F°.*

The intuition is as follows: if $F(\mu^n)$ converges to a distribution other than F°, then the cost of information $C(\mu^n)$ converges to a positive constant, and the auction price converges to value (see for example, Atakan and Ekmekci (2021), Lemma 2.2). However, if the price converges to value, then no bidder would be willing to bear a positive cost of information as the value from winning an object in the auction converges to zero. Lemma 2 highlights the main tension in the model: the auction price can only incentivize a small investment in information as the market grows large.

We now link equilibrium behavior to the equilibrium price by deriving the expected equilibrium prices in the two states. In the lemma presented below, we show that the expected profit in state v_1 converges to the expected loss in state v_0 as the market grows arbitrarily large.

Lemma 3. *Along any equilibrium sequence, the expected payoff of each bidder converges to zero, and we have* $1 - \lim \mathbb{E}[P^n|v_1] = \lim \mathbb{E}[P^n|v_0]$.

The reasoning for Lemma 3 is as follows: Lemma 2 implies that almost all bidders are approximately uninformed, that is, almost all bidders draw a type arbitrarily close to $1/2$. Since almost all the bidders are asymptotically uninformed, there is an uninformed bid that wins with probability converging to one, and the payoff to this bid converges to $(1 - \lim \mathbb{E}[P^n|v_1])/2 - \lim \mathbb{E}[P^n|v_0]/2$. Similarly, there is an uninformed bid that loses with probability converging to one, and the payoff to this bid converges to zero. Since an uninformed type must be indifferent between these two bids, we find that $1 - \lim \mathbb{E}[P^n|v_1] = \lim \mathbb{E}[P^n|v_0]$.

In the next lemma, we argue that the cost-accuracy ratio determines the expected loss in state v_0 and, therefore, the expected profit in state v_1.

Lemma 4. *If* $CA < 1/4$, *then* $1 - \lim \mathbb{E}[P^n|v_1] = 2CA$ *along any equilibrium sequence.*

We first provide some intuition for the measure of accuracy that we use. We then explain the argument for Lemma 4. Consider a bidder that chooses experiment F and bids according to her equilibrium strategy. This bidder draws a type $\theta > 1/2$ with probability $F(\{\theta > 1/2\}|v_i)$ in state v_i and wins with probability converging to one. This is because bidding is monotone in θ, and almost all bidders are asymptotically uninformed. A similar logic implies that the bidder loses with probability converging to one if she draws a type $\theta < 1/2$. Therefore, the payoff of this bidder converges to

$$\frac{\bar{F}(1/2|v_1)}{2}(1 - \lim \mathbb{E}[P^n|v_1]) - \frac{\bar{F}(1/2|v_0)}{2} \lim \mathbb{E}[P^n|v_0] - C(F).$$

Rewriting the equation displayed above by using the fact that the expected profit in state v_1 equals the expected loss in state v_0 we obtain the following expression,

$$\frac{1}{2}\underbrace{\left(\bar{F}(1/2|v_1) - \bar{F}(1/2|v_0)\right)}_{\text{Accuracy of } F}\ \underbrace{(1 - \lim \mathbb{E}[P^n|v_1])}_{\text{Divergence of price from value}}\ - C(F). \tag{3.1}$$

Therefore, the limit payoff to choosing experiment F only depends on the experiment's accuracy $A(F)$ and the experiment's cost. Moreover, the marginal value of accuracy equals the divergence of price from value.

We now turn to the argument for Lemma 4. Lemma 3 argued that the equilibrium payoff of any bidder converges to zero, or equivalently, the limit payoff to choosing any experiment $F \in \mathcal{F}$ is at most zero. Therefore, $1 - \lim \mathbb{E}[P^n|v_1] \leq 2CA_F$ for each $F \neq F^\circ$. Lemma 1 showed that bidders acquire information if $CA < 1/4$, that is, $C(F(\mu^n)) > 0$ for each n. We complete the argument for Lemma 4 by showing that $1 - \lim \mathbb{E}[P^n|v_1] \geq 2CA$. The intuition is easier to convey if the set of experiments is finite: If $1 - \lim \mathbb{E}[P^n|v_1] < 2CA$, then the payoff from choosing any experiment $F \neq F^\circ$ and therefore the payoff to any experiment $F \neq F^\circ$ in the support of μ^n is negative for sufficiently large n. However, since the set of experiments is finite and $F(\mu^n) \neq F^\circ$ for each n, the expected payoff of a bidder is negative for all sufficiently large n, leading to a contradiction. The formal argument is more involved because it also considers the case where the set of experiments is infinite, and the accuracy of the experiments in the support of μ^n converge to zero as n grows large.

We present our main theorem below.

Theorem 1. *The auction Γ^n has a symmetric equilibrium (μ^n, H^n) for each n. Information is aggregated if and only if $CA = 0$.*

Proof. The proof of equilibrium existence is in the appendix. If $CA = 0$, then Lemma 4 implies that $\lim \mathbb{E}[P^n|v_1] = 1$ and $\lim \mathbb{E}[P^n|v_0] = 0$. Therefore, P^n converges in probability to V and information is aggregated. If $CA \in (0, 1/4)$, then Lemma 4 implies that $\lim \mathbb{E}[P^n|V] \neq V$, therefore P^n does not converge in probability to V and thus information is not aggregated. The remaining case involves $CA \geq 1/4$. The proof Theorem 2 in the appendix shows that the price converges to $1/2$ in both states, i.e., the price is uninformative, if $CA \geq 1/4$. □

We can convey the main implications of this theorem by looking at two distinct cases.

Case 1: The cost-accuracy ratio is equal to CA_{F^*} for some $F^* \neq F^\circ$. For instance, if there are a finite number of experiments in $\mathcal{F}$, then $CA = CA_{F^*}$. In this case, information is not aggregated because $CA_{F^*} > 0$. Therefore, a sufficiently rich set of experiments is necessary for information aggregation.

Lemma 4 also provides insights about equilibrium behavior with finitely many experiments. The equilibrium experimentation strategy entails mixing between experiment F^* and the uninformative experiment for all sufficiently large n. This is because Lemma 4 and Equation (3.1) together imply that the limit payoff from choosing any experiment other than F^* is negative.

Case 2: There is an infinite number of experiments in $\mathcal{F}$, but there is no $F \in \mathcal{F} \setminus \{F^\circ\}$ with $CA = CA_F$. This implies that $CA = \liminf_{F \to F^\circ} CA_F$. In this case, information is aggregated if and only if $\liminf_{F \to F^\circ} CA_F = 0$. In other words, the behavior of the cost-accuracy ratio around the uninformative experiment (or equivalently, the marginal cost of accuracy evaluated at zero) determines whether information is aggregated if the set of experiments is sufficiently rich. We provide a detailed analysis of equilibrium behavior for this case in Section 3.2.

We now turn our attention to quantifying the information content of the price when information is not aggregated. Recall that the information content of the price is given by the distribution of an outsider's posterior upon observing the equilibrium price. Below we show that the distribution of the outsider's posterior and the distribution of the equilibrium price coincide at the limit.

Lemma 5. *If the price P^n converges in distribution to a random variable P along an equilibrium sequence, then the posterior $\Pr\{v_1 | P^n\}$ converges in distribution to P also.*

This lemma implies that we can simply concentrate on the price distribution when studying the information content of the auction price. The intuition is as follows: In equilibrium, a type θ bidder submits a bid equal to her posterior conditional on the event that she wins an object at a price equal to her bid, that is, $b = \Pr\{v_1 | b = Y^{n-1}(\lfloor \kappa n \rfloor), \theta\}$. The posterior of an outsider upon observing a price equal to b is given by $\Pr\{v_1 | b = Y^n(\lfloor \kappa n \rfloor + 1)\}$. Comparing the two posteriors, we find that they differ by the bidder's type θ and the information that the bidder won at a price equal to her bid, which appears as a one person difference in the order statistic calculation. As uninformed bidders set the price, the bidder's type adds no information at the limit. Moreover, the one-person difference in the order statistic calculation also disappears as n grows large. In other words, the information a bidder uses in determining her bid is the same as the information an outsider obtains from observing a price that equals this bid.

In order to present our result, we introduce some definitions. For any sequence of experimentation strategies μ^n, define

$$\Delta := \lim \sqrt{\frac{n}{(1-\kappa)\,\kappa}} \left(\bar{F}(1/2 | V = 1; \mu^n) - \bar{F}(1/2 | V = 0; \mu^n) \right), \qquad (3.2)$$

whenever this limit exists. Recall that $\bar{F}(1/2|V = 1; \mu^n) - \bar{F}(1/2|V = 0; \mu^n)$ is the accuracy of the experiment chosen by each bidder in equilibrium. The constant Δ is an aggregate measure of the accuracy of the information available in the market, asymptotically. For any $p \in (0,1)$, $\Delta \geq 0$, and $V \in \{0,1\}$ let

$$\zeta(p, \Delta, V) := \frac{\ln\left[\frac{p}{1-p}\right]}{\Delta} + \Delta\frac{1-2V}{2}. \tag{3.3}$$

Let $\Delta^*(CA)$ denote the unique solution (if one exists) to the equation

$$\int_{[0,1]} p d\Phi\left(\zeta(p, \Delta, 0)\right) = 2CA \tag{3.4}$$

where the integral is taken with respect to p and Φ is the standard normal cumulative distribution function.

Theorem 2. *The equilibrium price P^n converges in distribution to a random variable P along any sequence of equilibria.*

i. If $CA \geq 1/4$, then $P = 1/2$ and the price is uninformative.

ii. If $CA \in (0, 1/4)$, then $\Pr\{P \leq p|V\} = \Phi\left(\zeta\left(p, \Delta^*(CA), V\right)\right)$ for all $p \in [0,1]$ and the Blackwell informativeness of the price is decreasing in CA.

Theorem 1 and 2 together fully characterize the limit price distribution. Theorem 1 already covered the case where $CA = 0$. Theorem 2 provides a closed-form solution for the limit price distribution if $CA < 1/4$ and shows that the price is uninformative if $CA \geq 1/4$.

Lemma 5 implies that the posterior log-likelihood ratio of an outsider that observes the price is given by $\ln\left[P/\left(1-P\right)\right]$. Focusing on the distribution of the posterior log-likelihood ratio instead of the price distribution makes it easier to interpret Theorem 2. A change of variable calculation implies that the log-likelihood ratio $\ln\left[P/\left(1-P\right)\right]$ has a normal distribution with means $\Delta^2/2$ and $-\Delta^2/2$ in states 1 and 0, respectively, and has standard deviations equal to Δ in both states.

The constant Δ, defined in Equation (3.2), measures the aggregate accuracy of the information dispersed among bidders. If $\Delta = \infty$, then the information dispersed among the bidders is highly accurate, and the equilibrium price aggregates information. Theorem 1 showed that this occurs if and only if $CA = 0$. Conversely, if $\Delta = 0$, then the aggregate accuracy of the information in the market is low, and the equilibrium price equals $1/2$, that is, the price is uninformative.

Theorem 2 further argues that this occurs if and only if $CA \geq 1/4$. For intermediate values of Δ (i.e., $\Delta \in (0, \infty)$) the informativeness of price is increasing in Δ. To see this, note that the difference in the means of the log-likelihood ratios across the two states is given by Δ^2 while the standard deviation is equal to Δ. Therefore, increasing Δ increases the distance between the two means measured in standard deviations, i.e., the price becomes more informative.

For each value of Δ, the expected price in state 0 is given by $\int_{[0,1]} pd\Phi\left(\zeta(p, \Delta, 0)\right)$. The equilibrium value for Δ must satisfy $\mathbb{E}[P|V = 0; \Delta] = 2CA$ by Lemma 4. Therefore, the equilibrium value for Δ, which we denote as $\Delta^*(CA)$, is determined by Equation (3.4). In the proof of Theorem 2, we show that the expected price in state 0, given on the left hand side of Equation (3.4), is decreasing in Δ. The expected price is equal to 0 and 1/2, if $\Delta = \infty$ and $\Delta = 0$, respectively. Therefore, the equation has a unique solution if the cost-accuracy ratio is no more than 1/4, and this solution is decreasing in the cost-accuracy ratio. As the aggregate accuracy of the information in the market as measured by Δ is decreasing in the cost-accuracy ratio, the informativeness of the auction price is also decreasing in the cost-accuracy ratio.

The functional form of the limit price distribution is a consequence of a version of the central limit theorem, and the intuition is as follows: In state 1, the auction clears at the κth quantile of the bid distribution with probability converging to one by the law of large numbers. Consider a bid p whose quantile is z standard deviations away from the κth quantile of the bid distribution in state 1, where the standard deviation is given by $\sqrt{(1-\kappa)\,\kappa/n}$. This bid is $z + \Delta$ standard deviations away from the κth quantile of the bid distribution in state 0. Moreover, the central limit theorem implies that the posterior likelihood ratio after observing a price equal to p is given by $\phi(z)/\phi\left(z+\Delta\right)$. Therefore, $\ln\left(p(z)/\left(1-p(z)\right)\right) = \ln\left(\phi(z)/\phi\left(z+\Delta\right)\right) = \Delta z - \Delta/2$. The limit price distribution is then obtained by observing that the limit price is less than or equal to $p\left(z\right)$ with probability $\Phi(z)$ and $\Phi(z+\Delta)$ in states 1 and 0, respectively.

3.2. Equilibrium Characterization with Posterior Separable Information Costs. In this subsection, we assume that the set of experiments is sufficiently rich, and the cost of information is posterior separable. We show that information aggregates if and only if the cost function is differentiable at the prior (Corollary 1) and we derive the equilibrium type distribution for each n (Theorem 3). The type distribution presented in this subsection together with the equilibrium bidding strategies presented in Lemma 1 provide a complete characterization of equilibrium behavior.

We begin by formally stating our two assumptions.

Definition 3. The set of experiments is **rich** if there exists $\theta' < 1/2$ and $\theta'' > 1/2$ such that any Bayesian plausible $F \in \Delta[\theta', \theta'']$ is an element of $\mathcal{F}$.

Note that richness does not require $\mathcal{F}$ to contain particularly informative experiments. Rather, richness requires that if a particular experiment is available, all less informative experiments are also available. In other words, bidders can flexibly choose the accuracy of their information.

Definition 4. The cost function $C : \mathcal{F} \to \mathbb{R}_+$ is **posterior separable** if

$$C(F) = \int_{[0,1]} c(\theta) dF(\theta)$$

for each $F \in \mathcal{F}$ where $c : [0, 1] \to \mathbb{R}$ is a convex function. We say that a posterior separable cost function is differentiable if c is differentiable.[9]

Posterior separable cost functions include the mutual information cost function (see Sims (2003; 2010)), log-likelihood ratio information cost function (see Pomatto et al. (2020)), and the quadratic information cost function (see Example 1 further below). A posterior separable cost function satisfies two key properties that we use in our arguments: 1) It is convex, 2) It is Blackwell monotone, that is, if experiment F is Blackwell more informative than experiment F', then F costs more than F'. The cost function C is Blackwell monotone because c is convex.

In the following lemma, we compute the cost-accuracy ratio. In the statement of the lemma, we denote the left and right derivatives of a convex function $c : [0, 1] \to \mathbb{R}$ by $c'_-(\theta)$ and $c'_+(\theta)$, respectively.[10]

Lemma 6. *If the set of experiments is rich and the cost function is posterior separable, then* $CA = \left(c'_+(1/2) - c'_-(1/2)\right)/4$.

Proof. Consider a symmetric binary experiment that generates posteriors $q > 1/2$ and $1 - q$ with equal probability. The cost-accuracy ratio of this experiment is given by

$$CA_q = \frac{c(q)/2 + c(1-q)/2}{2q - 1}.$$

[9]Note that this is not the usual definition of differentiability (e.g. Frechet or Gateaux differentiability) for the functional $C : \Delta[0, 1] \to \mathbb{R}$.

[10]These directional derivatives exist since c is convex.

Richness implies that all symmetric binary experiments with $q \leq \theta''$ and $1-q \geq \theta'$ are in $\mathcal{F}$. Therefore,

$$CA \leq \lim_{q \to 1/2} \frac{c(q)/2 + c(1-q)/2}{2q-1} = \lim_{q \to 1/2} \frac{c'_+(1/2) - c'_-(1/2)}{4}.$$

For any $F \in \mathcal{F}$, we have

$$\begin{aligned} CA_F &= \frac{\int_{[0,1]} c(\theta) dF(\theta)}{\bar{F}(1/2|v_1) - \bar{F}(1/2|v_0)} \\ &\geq \frac{\int_0^{1/2} (\theta - 1/2)\, c'_-(1/2)\, dF(\theta) + \int_{1/2}^1 (\theta - 1/2)\, c'_+(1/2)\, dF(\theta)}{\int_{1/2}^1 2\theta dF(\theta) - \int_{1/2}^1 2(1-\theta)\, dF\theta} \end{aligned}$$

where the inequality follows because c is convex. Moreover, Bayesian plausibility implies $\int_0^{1/2} (\theta - 1/2)\, dF(\theta) + \int_{1/2}^1 (\theta - 1/2)\, dF(\theta) = 0$. Therefore,

$$CA_F \geq \frac{\left(c'_+(1/2) - c'_-(1/2)\right) \int_{1/2}^1 (\theta - 1/2)\, dF(\theta)}{\int_{1/2}^1 4(\theta - 1/2)\, dF\theta} = \frac{c'_+(1/2) - c'_-(1/2)}{4}.$$

Since this is true for each F we have $CA \geq \left(c'_+(1/2) - c'_-(1/2)\right)/4$. □

Corollary 1. *Assume that the set of experiments is rich and the cost function is posterior separable. Information is aggregated if and only if the cost function is differentiable at $\theta = 1/2$.*

Proof. Lemma 6 showed that $4CA = c'_+(1/2) - c'_-(1/2)$ and therefore information is aggregated if and only if $c'_+(1/2) = c'_-(1/2)$ by Theorem 1. □

We now introduce some notation that we need to present our equilibrium characterization. Let

$$b(\theta) := \frac{\theta G(\theta|v_1)}{\theta G(\theta|v_1) + (1-\theta)\, G(\theta|v_0)},$$

where

$$G(\theta|v_i) := \binom{n-1}{1} \binom{n-2}{\lfloor \kappa n \rfloor - 1} f(\theta|v_i)\, (1 - F(\theta|v_i))^{\lfloor \kappa n \rfloor - 1}\, F(\theta|v_i)^{n-1-\lfloor \kappa n \rfloor}.$$

The function $b(\theta)$ is the unique bid that type θ submits if the type distribution F is atomless, and $G(\theta|v_i)$ is the density of the $\lfloor \kappa n \rfloor$th highest out of n signals in state v_i.

Theorem 3. *Assume that $\mathcal{F}$ is rich, the cost function is posterior separable, and the function c is strictly convex. In any symmetric equilibrium (μ, H), the support of the type distribution F is an interval $\left[\underline{\theta}, \bar{\theta}\right]$ and the type distribution features an atom at $\theta \in \left(\underline{\theta}, \bar{\theta}\right)$ if and only if the function c is not differentiable at θ. Moreover, the type distribution satisfies the following differential equation*

$$(1 - b(\theta))\, G(\theta|v_1) + b(\theta)G(\theta|v_0) = c''(\theta) \tag{3.5}$$

for almost every $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$.

The main qualitative finding of the theorem is that the support of the equilibrium type distribution is an interval. In other words, there is a continuum of active types in equilibrium. Moreover, the type distribution is continuous whenever the function c is differentiable. The theorem also provides a partial characterization of the equilibrium type distribution through a differential equation (Equation (3.5)). We first give some intuition for our characterization. At the end of this subsection, we illustrate how one can solve for an equilibrium using Equation (3.5) through a simple example

Much of our analysis in this subsection rests on a key result in which we show that any bidder is indifferent between the equilibrium type distribution F and any other experiment $G \in \mathcal{F}$ with finite support that puts probability mass only on types in the support of the equilibrium type distribution. In other words, if $\text{supp } G \subset \text{supp } F$, then $U(F, H) = U(G, H)$. In particular, a bidder is indifferent between F and any binary experiment that splits the prior into two types $\theta^*, \theta \in \text{supp } F$. To see this, pick any experiment $G \in \mathcal{F}$ with $\text{supp } G \subset \text{supp } F$ and consider the following maximization problem:

$$\max_{a \in [0,1]} U\left((1-a)\, F + aG, H\right).$$

This maximization problem is solved at $a = 0$ because the equilibrium type distribution F is chosen optimally from $\mathcal{F}$, given that all players use the equilibrium bidding strategy H in the continuation game. The first-order condition for optimality implies

$$\frac{d}{da} U\left((1-a)\, F + aG, H\right)|_{a=0} = 0.$$

Recall that $U(F, H) = \int_\Theta \left(u^H(H(\theta), \theta) - c(\theta)\right) dF(\theta)$ and therefore $U\left((1-a)\, F + aG, H\right) = aU\left(F, H\right) + (1-a)\, U\left(G, H\right)$. Calculating this first-order condition explicitly, we find that $U(F, H) = U(G, H)$.

Let $\bar{\theta}$ and $\underline{\theta}$ denote the upper and lower bounds of the support of the equilibrium type distribution. We now argue that the type distribution F is strictly increasing over $\left[\underline{\theta}, \bar{\theta}\right]$, i.e., its support is an interval. Assume to the contrary that there are two types θ_1 and θ_2 in the support of the equilibrium type distribution such that F is flat over the interval (θ_1, θ_2). Note that there is an experiment G in $\mathcal{F}$ that puts positive probability only on θ_1, θ_2, and possibly an additional type $\theta^* \in \text{supp } F$.[11] By the argument given above, we know that $U(F, H) = U(G, H)$. We will construct a type distribution $F' \in \mathcal{F}$ that is Blackwell less informative than G and consequently cheaper than G. We will then use this type distribution to show that there is a profitable deviation from an equilibrium where distribution F is chosen with probability one.

Let b_1 denote the highest equilibrium bid submitted by type θ_1 and let b_2 denote the lowest equilibrium bid submitted by type θ_2. Lemma 1 argued that the bid distribution is atomless. Moreover, the bid distribution is flat between b_1 and b_2 because there are no types between θ_1 and θ_2 and because the bid distribution is monotone (Lemma 1). Therefore, in state v_1 the equilibrium payoff from bid b_1 is equal to the equilibrium payoff from bid b_2. Similarly, $u(b_1, v_0) = u(b_2, v_0)$.

Consider an alternative type distribution $F' \in \mathcal{F}$ that is identical G except that F' merges the two types θ_1 and θ_2 into a single type $\hat{\theta}$. More precisely, F' generates the posterior $\hat{\theta} = \left(F\left(\{\theta_1\}\right)\theta_1 + F\left(\{\theta_2\}\right)\theta_2\right)/F\left(\{\theta_1, \theta_2\}\right)$ with probability $F\left(\{\theta_1, \theta_2\}\right)$. Note that G is more informative than F' therefore $C(G) > C(F')$. However, choosing experiment F' and bidding b_2 after drawing $\hat{\theta}$ is a profitable deviation because $u(b_2, v_i) = u(b_1, v_i)$ for $i = 0, 1$ implies that $U(F', H) > U(G, H) = U(F, H)$.

We will now establish Equation (3.5). Let $G_{\underline{\theta},\theta}$ denote the binary experiment that generates posteriors $\underline{\theta}$ and $\theta \in [1/2, \bar{\theta}]$. Note that $U(G_{\underline{\theta},\theta}, H) = U(F, H)$ for each $\theta \in [1/2, \bar{\theta}]$. Therefore,

$$\frac{d}{d\theta} U(G_{\underline{\theta},\theta}, H) = 0.$$

An explicit calculation gives

$$U(G_{\underline{\theta},\theta}, H) = \frac{\left(u(H(\theta), \theta) - c(\theta)\right)\left(1/2 - \underline{\theta}\right) + \left(u(H(\underline{\theta}), \underline{\theta}) - c(\underline{\theta})\right)\left(\theta - 1/2\right)}{\theta - \underline{\theta}}$$

[11] If $1/2 \notin (\theta_1, \theta_2)$, then we need the experiment G to put weight on an addition type θ^* in order for Bayesian plausibility to be satisfied.

and $dU(G_{\underline{\theta},\theta}, H)/d\theta = 0$ implies that

$$\underbrace{\frac{d}{d\theta}\left(u(H(\theta),\theta) - c(\theta)\right)}_{\text{Marginal Interim Payoff}-\text{Marginal Cost}} = D, \tag{3.6}$$

where $D = c(\underline{\theta}) - u(H(\underline{\theta}), \underline{\theta})$ is a constant independent of θ. We will use Equation (3.6) to show that 1) The type distribution is atomless under the assumption that c is differentiable, and 2) The equilibrium type density satisfies Equation (3.5).

Recall that $u(H(\theta), \theta) = \theta u(H(\theta), v_1) + (1-\theta)\, u(H(\theta), v_0)$. The envelope theorem, or equivalently, the fact that $H(\theta)$ is chosen optimally in equilibrium, implies that $\frac{d}{d\sigma} u^H(\sigma, \theta)|_{\sigma = H(\theta)} = 0$. Therefore, the marginal interim payoff of type θ is equal to $u(H(\theta), v_1) - u(H(\theta), v_0)$. If the type distribution has an atom at θ, then the function $u(H(\theta), v_1) - u(H(\theta), v_0)$ is discontinuous at θ. This implies that the marginal cost is also discontinuous at θ, contradicting the differentiability of c.

We now complete the sketch of the proof by deriving the differential equation that characterizes the equilibrium type distribution. The fact that the type distribution is continuous at each $\theta \in \left(\underline{\theta}, \bar{\theta}\right)$ and Lemma 1 together imply that each type θ submits a unique bid equal to $b(\theta)$. Therefore,

$$u(\theta, v_i) = \int_0^\theta \left(v_i - b(\theta')\right) G(\theta'|v_i) d\theta'.$$

Moreover, the convex function c is twice differentiable almost everywhere by Alexandrov's Theorem. Therefore, $c'(\theta) = \int_0^\theta c''(\theta) d\theta$. Substituting $\int_0^\theta \left(v_i - b(\theta')\right) G(\theta'|v_i) d\theta'$ for $u(\theta, v_i)$ and $\int_0^\theta c''(\theta) d\theta$ for $c'(\theta)$ in Equation (3.6) we obtain

$$\int_0^\theta \left(\left(1 - b(\theta')\right) G(\theta'|v_1) + b(\theta') G(\theta'|v_0) - c''(\theta')\right) d\theta' = D$$

for each θ. The righthand side of the expression above is constant, and the identity must hold for each θ. Therefore,

$$\left(1 - b(\theta)\right) G(\theta|v_1) + b(\theta') G(\theta|v_0) = c''(\theta)$$

for almost every θ, establishing Equation (3.5).

In what follows, we turn our attention to solving for an equilibrium under the assumptions of Theorem 3. Equation (3.5), which characterizes the equilibrium type distribution, can be expressed as a second-order differential equation. However, solving this differential equation is not, in general, straightforward. We will illustrate how to derive the equilibrium by focusing on a simple example where there are two bidders and one object. In this case, Equation (3.5) reduces to the following expression

$$f(\theta) = \frac{\theta^2 + (1-\theta)^2}{\theta(1-\theta)} c''(\theta), \tag{3.7}$$

because $b(\theta) = \theta^2 / \left(\theta^2 + (1-\theta)^2\right)$. Moreover, the type distribution satisfies $F(\bar{\theta}) - F(\underline{\theta}) = 1$ and Bayesian plausibility $\int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta = 1/2$. Given the density f, the Bayesian plausibility constraint and $F(\bar{\theta}) - F(\underline{\theta}) = 1$ together uniquely determine the endpoints of the type distribution's support (See Kim and Koh (2022) for a detailed argument). In the examples below, we explicitly solve for the unique equilibrium for two alternative specifications of the cost function. For both of these examples, we assume that $\mathcal{F} = \Delta([\theta', \theta''])$, where $\theta' \leq 0.38 < 0.68 \leq \theta''$.

Example 1. Suppose that the cost of information is given by the quadratic information cost function, i.e., $c(\theta) = (1/2 - \theta)^2$. In this case, $c''(\theta) = 2$. Integrating Equation (3.7) and solving for $\underline{\theta}$ and $\bar{\theta}$ delivers the equilibrium type distribution

$$F^*(\theta) = \begin{cases} -4\theta + 2\ln\frac{\theta}{1-\theta} + 2.91 & \text{if } \theta \in [0.38, 0.62], \\ 0 & \text{if } \theta < 0.38, \\ 1 & \text{if } \theta > 0.62. \end{cases}$$

Note that $\theta' \leq 0.38 < 0.68 \leq \theta''$ implies that $F^* \in \mathcal{F}$. In order to find the endpoints of the support, we used the fact that $\underline{\theta} = 1 - \bar{\theta}$ (because the density f is symmetric around $1/2$) and $F(\bar{\theta}) - F(\underline{\theta})$.

Example 2. Suppose that the cost of information is given by mutual information cost function, i.e., $c(\theta) = \theta \ln\theta + (1-\theta)\ln(1-\theta)$. In this case, $c''(\theta) = 1/\theta(1-\theta)$ and therefore

$$F^*(\theta) = \begin{cases} \frac{2\theta - 1}{\theta(1-\theta)} + 0.49 & \text{if } \theta \in [0.44, 0.56], \\ 0 & \text{if } \theta < 0.44, \\ 1 & \text{if } \theta > 0.56. \end{cases}$$

As in the previous example, $\theta' < 0.44 < 0.56 < \theta''$ implies that $F^* \in \mathcal{F}$.

3.3. Information aggregation with general cost functions. Corollary 1 provided a necessary and sufficient condition for information aggregation with posterior separable cost functions. In this subsection, we focus on more general cost functions and provide two conditions that are together sufficient for information aggregation.

The first condition requires that the cost function be convex with respect to mixtures with the uninformative experiment in a neighborhood of the uninformative experiment.

Definition 5. The cost function satisfies **non-redundancy** if there exists an open ball around the uninformative experiment $N(F^\circ)$ such that $C(\gamma F+(1-\gamma)F^\circ) \leq \gamma C(F)$ for each binary experiment $F \in N(F^\circ) \cap \mathcal{F}$ and $\gamma \in [0,1]$.

We call this condition non-redundancy because if the experiment $F' = \gamma F + (1-\gamma) F^\circ$ is more costly than $\gamma c(F)$, where $\gamma \in (0,1)$, then no decision-maker would choose experiment F'. This is because a decision-maker can create an identical distribution over posteriors at a cost equal to $\gamma c(F)$ by mixing F with the uninformative experiment. Any posterior separable cost function satisfies non-redundancy.

We introduce some notation to state the second condition. Given an experiment F, define a new experiment that is obtained by taking two independent draws from F and denote this experiment by $F \otimes F$.

Definition 6. The cost function satisfies **ρ-monotonicity** if there exists a ball $N(F^\circ)$ and a constant $\rho > 1$ such that $C(F \otimes F) \geq \rho C(F)$ for each symmetric binary experiment F such that $F \otimes F \in N(F^\circ) \cap \mathcal{F}$.

The experiment obtained by taking two independent draws from F is more informative than F. Intuitively, one would then expect $F \otimes F$ to cost more than F if the experiments are correctly priced. The condition introduced above strengthens this notion of monotonicity by requiring $C(F \otimes F)/C(F)$ to be uniformly bounded below by a constant greater than one in a neighborhood around the uninformative experiment. We note that twice continuously differentiable posterior-separable cost functions satisfy ρ-monotonicity.[12] See Pomatto et al. (2020) for a detailed discussion of these two conditions.

We now argue that a rich set of experiments, non-redundancy, and ρ-monotonicity are together sufficient for information aggregation. Choose a symmetric binary experiment in $\mathcal{F}$ that generates posteriors q and $1-q$ with equal probability. The experiment's likelihood ratio is given by $\lambda := q/(1-q)$ and we denote this experiment by $F(\lambda)$ since its likelihood ratio fully characterizes it. Taking two independent draws from the experiment $F(\sqrt{\lambda})$ generates posteriors equal to q, $1/2$, and $1-q$ with probabilities $x(\lambda)/2$, $1-x(\lambda)$, and $x(\lambda)/2$, respectively. In other

[12]More precisely, a continuously differentiable posterior separable cost function satisfies $\rho-$monotonicity if $\limsup_{\theta \downarrow 1/2} \frac{c'(\theta)-c'(1/2)}{\theta-1/2} < \infty$ or $\limsup_{\theta \uparrow 1/2} \frac{c'(\theta)-c'(1/2)}{\theta-1/2} < \infty$.

words, experiment $F(\sqrt{\lambda}) \otimes F(\sqrt{\lambda})$ is equivalent to the experiment generated by picking binary experiment $F(\lambda)$ with probability x and picking the uninformative experiment with the remaining probability. Therefore, the cost of experiment $F(\sqrt{\lambda}) \otimes F(\sqrt{\lambda})$ is equal to the cost of experiment $xF(\lambda) + (1-x)F^{\circ}$. The lemma below establishes a linear bound for the cost of experiment $F(\sqrt{\lambda})$ in terms of the cost of $F(\lambda)$.

Lemma 7. *If the set of experiments is rich and the cost function satisfied non-redundancy and ρ-monotonicity, then*

$$C\left(F\left(\sqrt{\lambda}\right)\right) \leq \frac{x(\lambda)}{\rho} C\left(F\left(\lambda\right)\right).$$

Proof. The ρ-monotonicity implies that $C\left(F(\sqrt{\lambda}) \otimes F(\sqrt{\lambda})\right) \geq \rho C\left(F\sqrt{\lambda}\right)$. Moreover, non-redundancy implies that $C\left(x(\lambda)F(\lambda) + (1-x(\lambda))F^{\circ}\right) \leq x(\lambda)C(F(\lambda)$. Hence, $c\left(F(\sqrt{\lambda}) \otimes F(\sqrt{\lambda})\right) \leq x(\lambda)C((F(\lambda))$ because

$$C\left(F(\sqrt{\lambda}) \otimes F(\sqrt{\lambda})\right) = C\left(xF(\lambda) + (1-x)F^{\circ}\right).$$

Therefore, $C\left(F\left(\sqrt{\lambda}\right)\right) \leq C((F(\lambda))\, x(\lambda)/\rho$. □

The following theorem shows that the two conditions outlined above are sufficient for information aggregation.

Theorem 4. *If the set of experiments is rich and the cost function satisfies non-redundancy and ρ-monotonicity, then information is aggregated along any equilibrium sequence.*

To prove this theorem, we construct a sequence of binary experiments in $\mathcal{F}$ and we show that the cost-accuracy ratio converges to zero along this sequence. The fact that information is aggregated then follows from Theorem 1.

The construction works as follows: Choose a sequence of symmetric binary experiments $\left\{F\left(\lambda^{1/2^j}\right)\right\}_{j=0}^{\infty}$ contained in the set of available experiments such that the first experiment in the sequence, $F(\lambda)$, is close to the uninformative experiment, that is, λ is close to one. The experiments in the sequence are decreasing in informativeness and the sequence converges to the uninformative experiment. Moreover, the accuracy of experiment $F(\lambda^{1/2^j})$ is equal to $\left(\lambda^{1/2^j}-1\right)/\left(\lambda^{1/2^j}+1\right)$. Therefore, the accuracy of experiment $F(\lambda^{1/2^j})$ converges to zero at the order of $1/2^j$.

The linear bound established in Lemma 7 implies that $C(F(\sqrt{\lambda})) \leq yC(F(\lambda))$, where $y := x(\lambda)/\rho$. The fact that we chose λ close to one implies that y is a positive constant strictly less than $1/2$. The same logic establishes that

$$C\left(F(\lambda^{1/2^j})\right) \leq yC\left(F\left(\lambda^{1/2^{(j-1)}}\right)\right)$$

for each $j > 1$. Therefore, proceeding recursively, we find $C\left(F(\lambda^{1/2^j})\right) \leq y^j C\left(F(\lambda)\right)$. In other words, the cost of the experiment $F\left(\lambda^{1/2^j}\right)$ converges to zero at the order of y^j. This implies that the cost-benefit ratio of experiment $F\left(\lambda^{1/2^j}\right)$ also converges to zero as $j \to \infty$. This is because 1) cost converges to zero at rate y^j, 2) the accuracy converges to zero at rate $1/2^j$, and 3) $y < 1/2$.

Remark 1. In contrast to Corollary 1, Theorem 4 provides only a sufficient condition for information aggregation. In fact there are differentiable posterior separable cost functions that do not satisfy $\rho-$monotonicity. Such an example is given below:

$$c(\theta) = \begin{cases} (\theta - 1/2)^{3/2} & \text{if } q \geq 1/2 \\ (1/2 - \theta)^{3/2} & \text{if } q < 1/2 \end{cases}$$

This cost function is differentiable at $1/2$ and, therefore, information is aggregated by Corollary 1. However, it can be readily verified that

$$\lim_{F \to F^\circ} C(F \otimes F)/C(F) = 1$$

for each symmetric binary experiment F.

4. Conclusion

Understanding frictions that hinder information aggregation is a central economic question studied by a vast literature and costly information is one such friction. In this paper, we studied information aggregation in a large common-value auction where information is costly. Our main result provided a necessary and sufficient condition under which information aggregates. In particular, we identified the cost-accuracy ratio as the parameter that determines the information content of the auction price. Moreover, we argued that the informativeness of the price is Blackwell decreasing in the cost-accuracy ratio. Our approach allowed us to conclude that information can aggregate in a large market under mild conditions even when information is costly.

Our characterization of the equilibrium type distribution provided further insights concerning the distribution of information among the bidders in the auction.

In particular, we showed that each bidder acquires a vanishingly small amount of information in equilibria where information aggregates. In contrast, a vanishing fraction of bidders acquire substantial information while all other bidders remain uninformed in equilibria where information does not aggregate.

In this paper, our focus was on the demand for information in a particular market and we summarized the supply of information by an information cost function. An interesting avenue for future research is to model the supply of information explicitly.

A. Appendix

Below we prove Lemma 2 by showing that $F(\mu^n)$ converges in distribution to F°.

Proof of Lemma 2. The facts that bidding is monotone and the bid distribution is atomless together imply that $P^k \to V$ in probability if and only if $\lim \sqrt{n}\left(\bar{F}(1/2|v_1;\mu^n) - \bar{F}(1/2|v_0;\mu^n)\right) \to \infty$ by Atakan and Ekmekci (2021), Lemma 2.2.

Pick any convergent subsequence $F(\mu^n) \to \hat{F}$ in distribution, which exists by Helly's theorem. If $\hat{F}(\cdot) \neq F^\circ$, then $\lim \sqrt{n}\left(\bar{F}(1/2|v_1;\mu^n) - \bar{F}(1/2|v_0;\mu^n)\right) = \infty$, and therefore, $P^k \to V$ in probability. For each $\epsilon > 0$, the equilibrium payoff to strategy (μ^k, H^k) satisfies the following inequality:

$$U^k(\mu^k, H^k) \leq \frac{1}{2}\sum_{i=0}^{1}\left(\Pr\{P^k \leq v_i - \epsilon|V = v_i\}v_i + \epsilon\right) - C(\mu^k).$$

The fact that $P^k \to V$ in probability implies $\Pr\{P^k \leq v_i - \epsilon|V = v_i\} \to 0$ for each ϵ. Hence, $\limsup U^k(\mu^k, H^k) \leq -\lim C(\mu^k)$, as ϵ is arbitrary. However, $\lim C(\mu^k) > 0$ because c is continuous and $\hat{F} \neq F^\circ$. Therefore, equilibrium payoffs are negative for sufficiently large k, a contradiction. □

Below we prove Lemma 3. In particular, we show that $1 - \lim \mathbb{E}[P^n|v_1] = \lim \mathbb{E}[P^n|v_0]$ along any equilibrium sequence $\mathbf{H}$ along which these limits exist. Subsequently, we will prove that these limits exist along any equilibrium sequence. In particular, if $CB < 1/4$, then we show that $\lim \mathbb{E}[P^n|v_0] = CB$ in Lemma 4. If $CB \geq 1/4$, then we show that $1 - \lim \mathbb{E}[P^n|v_1] = \lim \mathbb{E}[P^n|v_0] = 1/2$ in Theorem 2.

Proof of Lemma 3. Pick a sequence of types θ^n and bids $b^n \in \operatorname{supp} H^n(\cdot|\theta^n)$ such that $\Pr(P^n \leq b^n|v_0) = \epsilon$. Notice that $\Pr(P^n \leq b^n|v_0) = \epsilon$ implies that $\Pr(P^n \leq b^n|v_1) \leq \epsilon$ because bidding is monotone in θ. Therefore $U^n(b^n|\theta^n) \leq \epsilon$. Consider an alternative strategy where type θ^n submits a bid equal to 1. The payoff to this strategy must be at most $U^n(b^n|\theta^n)$, that is,

$$\theta^n\left(1 - \mathbb{E}[P^n|v_1]\right) - (1 - \theta^n)\mathbb{E}[P^n|v_0] \leq \epsilon.$$

Moreover, $\Pr(P^n \leq b^n|v_0) = \epsilon$, $b^n \in \operatorname{supp}\ H^n(\theta^n)$, and $F(\mu^n) \overset{\mathcal{D}}{\to} F^\circ$ together imply that $\theta^n \to 1/2$. Therefore,

$$1 - \lim \mathbb{E}[P^n|v_1] - \lim \mathbb{E}[P^n|v_0] \leq 2\epsilon v_1$$

for each ϵ.

Pick a type θ^n and a bid $b^n \in \text{supp } H^n(\cdot|\theta^n)$ such that $H^n \{(\theta, b) : (\theta, b) \geq (\theta^n, b^n)|v_1\} = \kappa/2$. The fact that $F(\mu^n) \xrightarrow{\mathcal{D}} F^\circ$ implies that $\theta^n \to 1/2$. Notice that $\lim \Pr(P^n \leq b^n|v_1) = 1$. MLRP implies that $H^n \{(\theta, b) : (\theta, b) \geq (\theta^n, b^n)|v_0\} \leq \kappa/2$ for each n. Therefore, $\lim \Pr(P^n \leq b^n|v_0) = 1$. Moreover,

$$\begin{aligned} \theta^n \Pr\{P^n \leq b^n|v_1\} (1 - \mathbb{E}[P^n|P^n \leq b^n, v_1]) - \\ (1 - \theta^n) \Pr\{P^n \leq b^n|v_1\}\mathbb{E}[P^n|P^n \leq b^n, v_0] \to \\ 1 - \lim \mathbb{E}[P^n|v_1] - \lim \mathbb{E}[P^n|v_0] \geq 0 \end{aligned}$$

establishing the result. □

The probability that a particular bid b is pivotal (i.e., $Y^n(\lfloor \kappa n \rfloor + 1) = b$) can be approximated using the central limit theorem. If

$$\lim \frac{\lfloor \kappa n \rfloor - nH^n(\{b' \geq b\}|v_i))}{\sqrt{n\kappa(1-\kappa)}} = a,$$

then $Bi(\lfloor \kappa n \rfloor; n, H^n(\{b' \geq b\}|v_i)) \to \Phi(a)$ where Bi denotes the binomial cumulative distribution, that is, the probability of drawing at least $\lfloor \kappa n \rfloor$ successes out of n draws where the probability of success is equal $H^n(\{b' \geq b\}|v_i)$. Moreover, if we let $q = H^n(\{b' \geq b\}|v_i))$, then

$$bi(\lfloor \kappa n \rfloor; n, q) = \binom{n}{\lfloor \kappa n \rfloor} q^{\lfloor \kappa n \rfloor}(1-q)^{n-\lfloor \kappa n \rfloor} = \frac{1 + \delta_n(q)}{\sqrt{2\pi n\kappa(1-\kappa)}} \phi \left(\frac{\lfloor \kappa n \rfloor - nq}{\sqrt{\kappa(1-\kappa)n}} \right) \tag{A.1}$$

where bi denotes the binomial density; and $\lim_{n\to\infty} \sup_{q:|nq - \lfloor \kappa n \rfloor| < n^t} \delta_n(q) = 0$ for $t < 2/3$ (see Lesigne (2005, Proposition 8.2)). For any $b \in [v_0, v_1]$ define

$$z_i^n(b) := \frac{\lfloor \kappa n \rfloor - (n-1)\, H^n(\{b' \geq b\}|v_i)}{\sqrt{(n-1)\kappa(1-\kappa)}}.$$

Lemma 8. *Pick a sequence of bids and types*$\{(b^n, \theta^n)\}$ *with* $b^n \in$ *supp* $H^n(\cdot|\theta^n)$. *Assume that* $\lim z_i^n(b^n) = z_i < \infty$ *for* $i = 0, 1$ *and* $\lim l(\theta^n) = \rho$. *For any* $\delta > 0$, *there exists an N such that for all $n > N$ we have*

$$(1-\delta)\,\phi(z_1)/\phi(z_0) \leq l(Y^n(\lfloor \kappa n \rfloor + 1) = b^n) \leq (1+\delta)\,\phi(z_1)/\phi(z_0).$$

Therefore, $l(Y^n(\lfloor \kappa n \rfloor + 1) = b^n) \to \phi(z_1)/\phi(z_0)$.

Proof. A direct computation shows that

$$l(Y^n(\lfloor \kappa n \rfloor + 1) = b^n) = l(\theta^n)\frac{bi(\lfloor \kappa n \rfloor; n-1, H^n(\{b \geq b^n\}|v_1))}{bi(\lfloor \kappa n \rfloor; n-1, H^n(\{b \geq b^n\}|v_0))}.$$

Eq. (A.1) implies that for any $\delta > 0$, there exists an N such that

$$(1-\delta)\,\frac{\phi(z_1^n(b^n))}{\phi(z_0^n(b^n))} \leq \frac{bi(\lfloor \kappa n \rfloor; n-1, H^n(\{b \geq b^n\}|v_1))}{bi(\lfloor \kappa n \rfloor; n-1, H^n(\{b \geq b^n\}|v_0))} \leq (1+\delta)\,\frac{\phi(z_1^n(b^n))}{\phi(z_0^n(b^n))}$$

for all $n > N$. Our assumption that $\lim z_i^n(b^n) = z_i < \infty$ and $\lfloor \kappa n \rfloor/(n-1) \to \kappa$ together establish that $\lim \sqrt{n}|H^n(\{b \geq b^n\}|v_i) - \kappa| < \infty$ for $i = 0, 1$. The fact that $\phi(z_i^n(b))$ is a continuous functions of b implies that for any $\delta > 0$, there exists an N such that for all $n > N$ we have $\rho\,(1-\delta)\,\phi(z_1)/\phi(z_0) \leq l(Y^n(\lfloor \kappa n \rfloor + 1) = b^n) \leq \rho\,(1+\delta)\,\phi(z_1)/\phi(z_0)$, where $\rho = \lim l(\theta^n)$. Moreover, the fact that $F(\mu^n) \xrightarrow{\mathcal{D}} F^\circ$ implies $\rho = 1$. □

Lemma 9. *Suppose $CA < 1/4$. Along any equilibrium sequence (H^n, μ^n) we have*

$$\liminf \sqrt{\frac{n}{\kappa\,(1-\kappa)}}\left(\bar{F}(1/2|v_1;\mu^n) - \bar{F}(1/2|v_0;\mu^n)\right) > 0.$$

Proof. Suppose $\lim \sqrt{n}\,(F(1/2|v_1;\mu^n) - F(1/2|v_0;\mu^n)) = 0$ along a subsequence, which, abusing notation, is index again by n. We will argue that $P^n \xrightarrow{\mathcal{P}} 1/2$ in both states. However, then the argument provided in Lemma 1 implies that any bidder can improve upon her equilibrium payoff by deviating to an experiment

$$CA_F = C(F)/\left(\bar{F}(1/2|v_1) - \bar{F}(1/2|v_0)\right) < 1/4$$

for sufficiently large n leading to a contradiction.

We now argue that $\lim \sqrt{n}\,(F(1/2|v_1;\mu^n) - F(1/2|v_0;\mu^n)) = 0$ implies that $P^k \xrightarrow{\mathcal{P}} 1/2$. Pick a sequence of bids and types$\{(b^n, \theta^n)\}$ with $b^n \in \operatorname{supp} H^n(\cdot|\theta^n)$. Assume that $\lim z_i^n(b^n) = z_i < \infty$ for $i = 0, 1$. Our maintained assumption $\lim \sqrt{n}\,(F(1/2|v_1;\mu^n) - F(1/2|v_0;\mu^n)) = 0$ implies that $z_1 = z_0$. Therefore, $\lim l(Y^n(\lfloor \kappa n \rfloor + 1) = b^n) = 1$ by Lemma 8. However, then $b^n \to 1/2$. Moreover, the price clears at a bid with $z_i < \infty$ with probability converging to one. Therefore, $P^n \xrightarrow{\mathcal{P}} 1/2$ in both states. □

Lemma 10. *Along any equilibrium sequence* (μ^n, H^n)*, we have* $\lim U^n(\mu^n, H^n) = 0$.

Proof. Pick a type $\theta^n \in [1/2-\epsilon, 1/2+\epsilon]$ and a bid $b^n \in \text{supp } H^n(\theta^n)$ such that $\lim \Pr(P^n \leq b^n|v_1) = 1$. The limit payoff for this type is at most $2\epsilon v_1$ because $1 - \lim \mathbb{E}[P^n|v_1] = \lim \mathbb{E}[P^n|v_0]$ and because $\lim \mathbb{E}[P^n|v_1] \geq 0$. Moreover, for any $\theta, \theta' \in [1/2-\epsilon, 1/2+\epsilon]$, $b \in \text{supp } H^n(\cdot|\theta)$, and $b' \in \text{supp } H^n(\cdot|\theta')$,

$$|u^n(b,\theta) - u^n(b',\theta')| \leq 2\epsilon.$$

Hence, $\lim u^n(b^n,\theta) \leq 4\epsilon$ for each $\theta \in [1/2-\epsilon, 1/2+\epsilon]$, $b^n \in \text{supp } H^n(\theta)$. Moreover, $F([1/2-\epsilon, 1/2+\epsilon]; \mu^n) \to 1$ because $F(\mu^n) \xrightarrow{\mathcal{D}} F^\circ$. Observing that ϵ is arbitrary establishes the result. □

Lemma 4 states if $CA < 1/4$, then $\lim_n \mathbb{E}[P^n|v_0] = CA$ along any equilibrium sequence $\mathbf{H}$. The argument for this lemma is below.

Proof of Lemma 4. We prove the lemma through two intermediate Lemmata presented below. The first lemma shows that $2CB_F \geq \limsup_n \mathbb{E}[P^n|v_0]$ for all $F \in \mathcal{F} \setminus \{F^\circ\}$ along any equilibrium sequence $\mathbf{H}$. The second lemma argues that if $\lim \sqrt{n}\,(F(1/2|v_1;\mu^n) - F(1/2|v_0;\mu^n)) > 0$ along an equilibrium sequence, then $2CB \leq \liminf_n \mathbb{E}[P^n|v_0]$ along the sequence $\mathbf{H}$. These two Lemmata together with Lemma 9 establish that if $CB < 1/4$, then $CB = \lim_n \mathbb{E}[P^n|v_0]$ along any equilibrium sequence. □

Lemma 11. *Along any equilibrium sequence* $\mathbf{H}$*, we have* $2CA_F \geq \limsup_n \mathbb{E}[P^n|v_0]$ *for all* $F \in \mathcal{F} \setminus \{F^\circ\}$.

Proof. Assume to the contrary that there exists an experiment $F > 0$ such that

$$2CA_F = \frac{2C(F)}{\bar{F}(1/2|v_1) - \bar{F}(1/2|v_0)} < \lim_n \mathbb{E}[P^n|v_0]$$

along some equilibrium sequence. Consider the strategy where a bidder purchases experiment F, bids 1 if $\theta > 1/2$, and bids 0 otherwise. The payoff from this strategy converges to $\left(\bar{F}(\theta|v_1) - \bar{F}(\theta|v_0)\right)\mathbb{E}[P|v_0]/2 - c(F) > 0$ because $1 - \lim \mathbb{E}[P^n|v_1] = \lim \mathbb{E}[P^n|v_0]$ by Lemma 3. However, this contradicts Claim 10, which showed equilibrium payoffs converge to zero. As this is true for any convergent sequence $\{\mathbb{E}[P^n|v_0]\}_n$ we have $2CA_F \geq \limsup_n \mathbb{E}[P^n|v_0]$ for all $F \in \mathcal{F} \setminus \{F^\circ\}$. □

Lemma 12. *Along any equilibrium sequence, if*

$$\lim \sqrt{n}\left(\bar{F}(1/2|v_1;\mu^n) - \bar{F}(1/2|v_0;\mu^n)\right) > 0,$$

then

$$CA \leq \liminf_n \mathbb{E}[P|v_0].$$

Proof. Pick a subsequence, which abusing notation we index by n, such that $F(\mu^n) \neq F^\circ$ for each n and $\lim CB_{F(\mu^n)}$ exists. Note that such a subsequence exists by assumption and $\lim CA_{F(\mu^n)} \geq CA$. Assume to the contrary of the claim that $\lim CA_{F(\mu^n)} \geq CA > \liminf_n \mathbb{E}[P|v_0]$.

Choose $\epsilon > 0$. Pick a type θ^n and a bid $b^n \in \text{supp } H^n(\theta^n)$ such that $H^n\{(\theta,b) : (\theta,b) \geq (\theta^n,b^n)\} = \kappa - \epsilon$. Pick a type $\hat{\theta}^n$ and a bid $\hat{b}^n \in \text{supp } H^n(\hat{\theta}^n)$ such that $H^n\left\{(\theta,b) : (\theta,b) \geq (\hat{\theta}^n,\hat{b}^n)\right\} = \kappa + \epsilon$. The equilibrium payoff can be expressed as follows

$$\begin{aligned} U^n(H^n,\mu^n) = & \\ &\int_{\theta^n}^1 \int_{v_0}^{v_1} \theta \Pr\{P^n \leq b|v_1\}\left(1 - \mathbb{E}[P^n|P^n \leq b, v_1]\right) \\ &\quad - (1-\theta)\Pr\{P^n \leq b|v_0\}\mathbb{E}[P^n|P^n \leq b, v_0])dH^n(b|\theta)dF(\theta;\mu^n) \\ &+ H^n\left(\left\{(\theta,b) : \theta \in [\hat{\theta}^n,\theta^n), b \geq \hat{b}^n\right\}\right)\mathbb{E}\left[u^n(b|\theta)|\theta \in [\hat{\theta}^n,\theta^n), b \geq \hat{b}^n\right] + \\ &H^n\left(\left\{(\theta,b) : \theta \leq \hat{\theta}^n, b < \hat{b}^n\right\}\right)\mathbb{E}\left[u^n(b|\theta)|\theta \leq \hat{\theta}, b < \hat{b}^n\right] - C(\mu^n) \quad \text{(A.2)} \end{aligned}$$

where

$$\begin{aligned} &\mathbb{E}\left[u^n(b,\theta)|\theta \in [\hat{\theta}^n,\theta^n), b \geq \hat{b}^n\right] = \\ &\int_{\left\{(\theta,b):\theta\leq\hat{\theta}^n, b<\hat{b}^n\right\}} \theta \Pr\{P^n \leq b|v_1\}\left(1 - \mathbb{E}[P^n|P^n \leq b, v_1]\right) - \\ &(1-\theta)\Pr\{P^n \leq b|v_0\}\mathbb{E}[P^n|P^n \leq b, v_0])\frac{dH^n}{H^n\left(\left\{(\theta,b) : \theta \leq \hat{\theta}, b < \hat{b}^n\right\}\right)}. \end{aligned}$$

Notice that for each $(\theta,b) \geq (\theta^n,b^n)$, we have

$$\begin{aligned} &\theta \Pr\{P^n \leq b|v_1\}\left(1 - \mathbb{E}[P^n|P^n \leq b, v_1]\right) - (1-\theta)\Pr\{P^n \leq b|v_0\}\mathbb{E}[P^n|P^n \leq b, v_0]) \leq \\ &\theta \Pr\{P^n \leq b|v_1\}\left(1 - \mathbb{E}[P^n|P^n \leq b^n, v_1]\right) - (1-\theta)\Pr\{P^n \leq b|v_0\}\mathbb{E}[P^n|P^n \leq b^n, v_0]). \end{aligned}$$

Moreover, $\Pr\{P^n \le b|v_1\} \le \Pr\{P^n \le b|v_0\}$ implies

$$\theta \Pr\{P^n \le b|v_1\}\left(1 - \mathbb{E}[P^n|P^n \le b^n, v_1]\right)) - (1-\theta)\Pr\{P^n \le b|v_0\}\mathbb{E}[P^n|P^n \le b^n, v_0]) \le \theta\left(1 - \mathbb{E}[P^n|P^n \le b^n, v_1]\right)) - (1-\theta)\mathbb{E}[P^n|P^n \le b^n, v_0])$$

Substituting back into the equation (A.2), we find

$$\begin{aligned} U^n(H^n, \mu^n) \le (1 - \mathbb{E}[P^n|P^n \le b^n, v_1])) \int_{\theta^n}^1 \theta dF(\theta;\mu^n) - \\ \mathbb{E}[P^n|P^n \le b^n, v_0]) \int_{\theta^n}^1 (1-\theta) dF(\theta;\mu^n) + \\ + H^n\left(\left\{(\theta, b) : \theta \in [\hat{\theta}, \theta^n), b \ge \hat{b}^n\right\}\right) \mathbb{E}\left[u^n(b|\theta)|\theta \in [\hat{\theta}, \theta^n), b \ge \hat{b}^n\right] + \Pr\{P^n \le \hat{b}^n\} - C(\mu^n) \end{aligned}$$

Using the facts that $\int_{\theta^n}^1 \theta dF(\theta;\mu^n) = \bar{F}(\theta^n|v_1;\mu^n)/2$, and $\int_{\theta^n}^1 (1-\theta)\, dF(\theta;\mu^n) = \bar{F}(\theta^n|v_0;\mu^n)/2$ we obtain

$$\begin{aligned} U^n(H^n, \mu^n) \le \bar{F}(\theta^n;\mu^n)\left(\frac{F_1^n}{2\bar{F}(\theta;\mu^n)}(1 - P_1^n) - \frac{F_0^n}{2\bar{F}(\theta;\mu^n)} P_0^n\right) \\ + H^n\left(\left\{(\theta, b) : \theta \in [\hat{\theta}, \theta^n), b \ge \hat{b}^n\right\}\right) \mathbb{E}\left[u^n(b|\theta)|\theta \in [\hat{\theta}, \theta^n), b \ge \hat{b}^n\right] + \Pr\{P^n \le \hat{b}^n\} - C(\mu^n) \end{aligned}$$

where $F_i^n = \bar{F}(\theta^n|v_i, \mu^n)$ and $P_i^n = \mathbb{E}[P^n|P^n \le b^n, v_i]$. Note that if $\theta > \theta'$, $b \in \text{supp } H^n(\theta)$, and $b' \in \text{supp } H^n(\theta')$, then $u^n(b'|\theta') \le u^n(b'|\theta) \le u^n(b|\theta)$. Moreover, $H^n\left(\left\{(\theta, b) : \theta \in [\hat{\theta}, \theta^n), b \ge \hat{b}^n\right\}\right) \le 2\epsilon$. Consequently,

$$\begin{aligned} U^n(H^n, \mu^n) &\le \frac{F_1^n}{2}(1 - P_1^n) - \frac{F_0^n}{2} P_0^n + \\ &\frac{2\epsilon}{\bar{F}(\theta^n, \mu^n)}\left(\frac{F_1^n}{2}(1 - P_1^n) - \frac{F_0^n}{2} P_0^n\right) + \Pr\{P^n \le \hat{b}^n\} - C(\mu^n) \\ &= \frac{F_1^n + \epsilon_1^n}{2}(1 - P_1^n) - \frac{F_0^n + \epsilon_0^n}{2} P_0^n + \Pr\{P^n \le \hat{b}^n\} - C(\mu^n) \end{aligned}$$

where $\epsilon_i^n = 2\epsilon \frac{F_0^n}{\bar{F}(\theta^n, \mu^n)}$. A bidder can always choose experiment F° and submit a bid equal to b^n. The payoff from this strategy is equal to

$$\frac{\Pr\{P^n \le b^n|v_1\}(1 - P_1^n) - \Pr\{P^n \le b^n|v_0\} P_0^n}{2}.$$

The fact that $F(\mu^n) \neq F^\circ$ implies that

$$\begin{aligned}\frac{q_1^n(1-P_1^n) - q_0^n P_0^n}{2} &\leq U^n(H^n, \mu^n) \\ &\leq \frac{F_1^n + \epsilon_1^n}{2}(1 - P_1^n) - \frac{F_0^n + \epsilon_0^n}{2} P_0^n + \Pr\{P^n \leq \hat{b}^n\} - C(\mu^n).\end{aligned}$$

where $q_i^n = \Pr\{P^n \leq b^n | v_i\}$. Therefore,

$$1 - P_1^n \leq \frac{q_0^n - F_0^n - \epsilon_0^n}{q_1^n - F_1^n - \epsilon_1^n} P_0^n + \frac{2\Pr\{P^n \leq \hat{b}^n\} - 2c(\mu^n)}{q_1^n - F_1^n - \epsilon_1^n}.$$

This in turn implies that

$$\begin{aligned}&\frac{F_1^n}{2}(1 - P_1^n) - \frac{F_0^n}{2} P_0^n \leq \\ &\quad \frac{F_1^n}{2}\left(\frac{q_0^n - F_0^n - \epsilon_0^n}{q_1^n - F_1^n - \epsilon_1^n} P_0^n + \frac{2\Pr\{P^n \leq \hat{b}^n\} - 2c(\mu^n)}{q_1^n - F_1^n - \epsilon_1^n}\right) - \frac{F_0^n}{2} P_0^n \\ &\quad = \frac{(q_0^n F_1^n - q_1^n F_0^n)\, P_0^n}{2(q_1^n - F_1^n - \epsilon_1^n)} + F_1^n \frac{\Pr\{P^n \leq \hat{b}^n\} - C(\mu^n)}{q_1^n - F_1^n - \epsilon_1^n}.\end{aligned}$$

Consequently,

$$\begin{aligned}U^n(H^n, \mu^n) &\leq \frac{F_1^n}{2}(1 - P_1^n) - \frac{F_0^n}{2} P_0^n + \\ &\quad \frac{2\epsilon}{\bar{F}(\theta^n, \mu^n)}\left(\frac{F_1^n}{2}(1 - P_1^n) - \frac{F_0^n}{2} P_0^n\right) + \Pr\{P^n \leq \hat{b}^n\} - C(\mu^n) \\ &\leq \frac{(q_0^n F_1^n - q_1^n F_0^n)\, P_0^n}{2(q_1^n - F_1^n - \epsilon_1^n)} + F_1^n \frac{\Pr\{P^n \leq \hat{b}^n\} - C(\mu^n)}{q_1^n - F_1^n - \epsilon_1^n} + \\ &\frac{2\epsilon}{\bar{F}(\theta^n, \mu^n)}\left(\frac{(q_0^n F_1^n - q_1^n F_0^n)\, P_0^n}{2(q_1^n - F_1^n - \epsilon_1^n)} + F_1^n \frac{\Pr\{P^n \leq \hat{b}^n\} - c(\mu^n)}{q_1^n - F_1^n - \epsilon_1^n}\right) + \Pr\{P^n \leq \hat{b}^n\} - C(\mu^n) = \\ &\quad \frac{(q_0^n F_1^n - q_1^n F_0^n)\, P_0^n}{2(q_1^n - F_1^n - \epsilon_1^n)} + (q_1^n - \epsilon_1^n)\, \frac{\Pr\{P^n \leq \hat{b}^n\} - C(\mu^n)}{q_1^n - F_1^n - \epsilon_1^n} + \\ &\quad \frac{2\epsilon}{\bar{F}(\theta^n, \mu^n)}\left(\frac{(q_0^n F_1^n - q_1^n F_0^n)\, P_0^n}{2(q_1^n - F_1^n - \epsilon_1^n)} + F_1^n \frac{\Pr\{P^n \leq \hat{b}^n\} - C(\mu^n)}{q_1^n - F_1^n - \epsilon_1^n}\right)\end{aligned}$$

Individual rationality implies that $U^n(H^n, \mu^n) \geq 0$. Therefore, dividing the righthand side by $(F_1^n - F_0^n)/(q_1^n - F_1^n - \epsilon_1^n)$ we find

$$0 \leq \frac{(q_0^n F_1^n - q_1^n F_0^n)(P_0^n - v_0^n)}{2(F_1^n - F_0^n)} + (q^n - \epsilon_1^n) \frac{\Pr\{P^n \leq \hat{b}^n\} - C(\mu^n)}{F_1^n - F_0^n} + \frac{2\epsilon}{\bar{F}(\theta^n, \mu^n)} \left(\frac{(q_0^n F_1^n - q_1^n F_0^n)(P_0^n - v_0^n)}{2(F_1^n - F_0^n)} + F_1^n \frac{\Pr\{P^n \leq \hat{b}^n\} - C(\mu^n)}{F_1^n - F_0^n} \right)$$

Notice that $F_1^n - F_0^n > 0$ because $F(\mu^n) \neq F^\circ$ and $q_1^n - F_1^n - \epsilon_1^n > 0$ for sufficiently large n because q_1^n converges to one. Further below we will argue that $\Pr\{P^n \leq \hat{b}^n\}/(F_1^n - F_0^n) \to 0$ and $(q_0^n F_1^n - q_1^n F_0^n)/(F_1^n - F_0^n) \to 1$. Taking the limit as n goes to infinity we find

$$0 \leq \lim \frac{(P_0^n - v_0^n)}{2} - \lim (1 - \epsilon_1^n) \frac{C(\mu^n)}{F_1^n - F_0^n} + \frac{2\epsilon}{\bar{F}(\theta^n, \mu^n)} \left(\lim \frac{P_0^n}{2} + \lim F_1^n \frac{\Pr\{P^n \leq \hat{b}^n\} - C(\mu^n)}{F_1^n - F_0^n} \right)$$

The fact that ϵ is arbitrary implies that

$$0 \leq \lim \mathbb{E}[P^n | v_0] - \lim \frac{2C(\mu^n)}{F_1^n - F_0^n}.$$

However, $F_1^n - F_0^n = \int_{\theta^n}^1 2(2\theta - 1) dF(\theta; \mu^n) \leq \int_{1/2}^1 2(2\theta - 1) dF(\theta; \mu^n) = \bar{F}(1/2 | v_1; \mu^n) - \bar{F}(1/2 | v_0; \mu^n)$ because $\theta^n \geq 1/2$ for sufficiently large n. Therefore,

$$0 \leq \lim \mathbb{E}[P^n | v_0] - \lim \frac{2C(\mu^n)}{\bar{F}(1/2 | v_1; \mu^n) - \bar{F}(1/2 | v_0; \mu^n)}.$$

We now argue that $\sqrt{n} \Pr\{P^n \leq \hat{b}^n\}/\sqrt{n}(F_1^n - F_0^n) \to 0$.

Note that $\lim \sqrt{n}\,(F(1/2|v_1;\mu^n) - F(1/2|v_0;\mu^n)) > 0$ by assumption and $\Pr\{P^n \leq \hat{b}^n\} \leq e^{-\frac{\left(\frac{\epsilon}{\kappa+\epsilon}\right)^2 n(\kappa+\epsilon)}{2}}$ by Chernoff's inequality (see Janson et al. (2011, Theorem 2.1)). Therefore, the facts that

$$\sqrt{n} \Pr\{P^n \leq \hat{b}^n\}/\sqrt{n}(F_1^n - F_0^n) \leq \sqrt{n} e^{-\frac{\left(\frac{\epsilon}{\kappa+\epsilon}\right)^2 n(\kappa+\epsilon)}{2}} / \sqrt{n}(F_1^n - F_0^n),$$

$\sqrt{n} e^{-\frac{\left(\frac{\epsilon}{\kappa+\epsilon}\right)^2 n(\kappa+\epsilon)}{2}} \to 0$, and $\sqrt{n}(F_1^n - F_0^n) \to c > 0$ together establish the limit.

Finally, we argue that $(q_0^n F_1^n - q_1^n F_0^n)/(F_1^n - F_0^n) \to 1$. Chernoff's inequality implies that $q_i = \Pr\{P^n \leq b^n|v_i\} \geq 1 - e^{-\frac{\left(\frac{\epsilon}{\kappa+\epsilon}\right)^2 n(\kappa+\epsilon)}{3}}$. Therefore,

$$\frac{\left(1 - e^{-\frac{\left(\frac{\epsilon}{\kappa+\epsilon}\right)^2 n(\kappa+\epsilon)}{3}}\right) F_1^n - F_0^n}{F_1^n - F_0^n} \leq \frac{q_0^n F_1^n - q_1^n F_0^n}{F_1^n - F_0^n} \leq \frac{F_1^n - \left(1 - e^{-\frac{\left(\frac{\epsilon}{\kappa+\epsilon}\right)^2 n(\kappa+\epsilon)}{3}}\right) F_0^n}{F_1^n - F_0^n}$$

$$1 - F_1^n \frac{e^{-\frac{\left(\frac{\epsilon}{\kappa+\epsilon}\right)^2 n(\kappa+\epsilon)}{3}}}{F_1^n - F_0^n} \leq \frac{q_0^n F_1^n - q_1^n F_0^n}{F_1^n - F_0^n} \leq 1 + F_1^n \frac{e^{-\frac{\left(\frac{\epsilon}{\kappa+\epsilon}\right)^2 n(\kappa+\epsilon)}{3}}}{F_1^n - F_0^n}.$$

The fact that $\sqrt{n} e^{-\frac{\left(\frac{\epsilon}{\kappa+\epsilon}\right)^2 n(\kappa+\epsilon)}{3}} / \sqrt{n} \left(F_1^n - F_0^n\right) \to 0$ establishes the limit. □

A.1. Proof of Theorem 1, equilibrium existence. The fact that $\mathcal{F}$ is compact in the weak-star topology implies that $\Delta\left(\mathcal{F}\right)$ is also compact in the weak-star topology. For each $\mu \in \Delta(\mathcal{F})$, there is a unique symmetric equilibrium bid distribution $H_{F(\mu)}$ by Lemma 1. Moreover, for each $b \in \text{supp } H_{F(\mu)}(\theta)$ we have

$$b = \mathbb{E}[V|Y^{n-1}(\lfloor \kappa n \rfloor) = b, \theta_i = \theta)].$$

Claim 1. If μ' converges to μ in distribution, then $F(\mu')$ converges to $F(\mu)$ in distribution and $H_{F(\mu)}$ converges to $H_{F(\mu)}$ in distribution.

Proof. For each $x \in (0, 1)$, let $b(x, \mu)$ be such that $H_{F(\mu)}(\{b' \leq b(x, \mu)\} | v_1) = x$, let $y(x, \mu) = H_{F(\mu)}(\{b' \leq b(x, \mu)\} | v_0)$ and let $\theta(x, \mu) = \inf \{\theta' : F(\theta'|v_1) \geq x\}$. The equilibrium bidding strategy implies that

$$\frac{b(x, \mu)}{1 - b(x, \mu)} = \frac{\theta(x, \mu)^2}{(1 - \theta(x, \mu))^2} \frac{(1 - x)^{k-1} x^{n-k-1}}{(1 - y(x, \mu))^{k-1} y(x, \mu)^{n-k-1}}$$

If $\mu^n \to \mu$, then $\theta(x, \mu^n) \to \theta(x, \mu)$. This is because θ is the xth quantile of $F(\mu^n)$ and if $F(\mu^n) \to F(\mu)$ in distribution, then the quantiles also converge. Notice that

$$y(x, \mu^n) = \int_0^x \frac{(1 - \theta(z, \mu^n))}{\theta(z, \mu_n)} dz.$$

Therefore, if $\mu^n \to \mu$,then $y(x, \mu^n) \to y(x, \mu)$. Hence, $\frac{b(x,\mu^n)}{1-b(x,\mu^n)} \to \frac{b(x,\mu)}{1-b(x,\mu)}$, completing the proof. □

For each $\mu \in \Delta\left(\mathcal{F}\right)$, let

$$u(\theta, \mu) := \max_b \theta \int_0^b (1 - p) dH_{F(\mu)}\left(p|v_1\right) - (1 - \theta) \int_0^b p dH_{F(\mu)}\left(p|v_0\right)$$

and for each $(\hat{\mu}, \mu) \in \Delta(\mathcal{F}) \times \Delta(\mathcal{F})$ let

$$U(\hat{\mu}, \mu) := \int_{\mathcal{F}} \int_{[0,1]} u(\theta, \mu) d\hat{F}(\theta) d\hat{\mu}(\hat{F}).$$

Claim 2. The function u is continuous in θ and continuous in μ in the weak-star topology. Moreover, the function U is continuous in both arguments in the weak-star topology.

Proof. The properties of u follow from Berge's maximum theorem. The properties of U follow from the continuity of u. □

Define the correspondence $\Gamma : \Delta(\mathcal{F}) \to \Delta(\mathcal{F})$ as follows

$$\Gamma(\mu) = \arg \max_{\hat{\mu} \in \Delta(\mathcal{F})} U(\hat{\mu}, \mu).$$

This correspondence is upper-hemi continuous and compact valued by Berge's maximum theorem. This is because $\Delta(\mathcal{F})$ is compact and U is continuous in the weak-star topology. Moreover, Γ is also convex valued since U is a linear function of $\hat{\mu}$. Therefore, Γ has a fixed-point $\mu^* \in \mathcal{F}$ by Glicksberg's fixed point theorem. The information acquisition strategy μ^* together with the equilibrium bidding distribution $H_{F(\mu^*)}$ comprises a symmetric equilibrium for the auction.

A.2. Proof of Theorem 2.

Lemma 13. *If*

$$\frac{\sqrt{n}\left(\bar{F}(1/2|v_1; \mu^n) - \bar{F}(1/2|v_0; \mu^n)\right)}{\sqrt{\kappa(1-\kappa))}} \to \Delta \in (0, \infty),$$

then $P^n \xrightarrow{\mathcal{D}} P$, *$P$ is atomless with support* $[0,1]$, *and distribution function* $\Pr\{P \leq p\} = \Phi(\zeta(p, \Delta))$ *for all* $p \in [0,1]$.

Proof. For any $p \in (0,1)$, we will argue that

$$\lim z_0^n(p) = \zeta(p, \Delta, 0) = \frac{\ln\left[\frac{p}{1-p}\right]}{\Delta} + \frac{\Delta}{2}.$$

Pick a sequence of types $\{\theta^n\}$ such that $p \in H^n(\cdot|\theta^n)$. Pick a subsequence along which $\lim z_0^n(p)$ exists and denote the limit point by z. We have $p = \mathbb{E}[V|Y^{n-1}(\lfloor \kappa n \rfloor) = p, \theta_i = \theta^n)]$ for each n. Moreover, if $z \in (-\infty, \infty)$, then

$\theta^n \to 1/2$ by Lemma 8. Therefore, if $\lim z_0^n(p) = z$, then $\lim z_1^n(p) = z - \Delta$ because bidding is monotone and because $\theta^n \to 1/2$.

Lemma 8 implies that

$$\lim \mathbb{E}[V|Y^{n-1}(\lfloor \kappa n \rfloor) = p, \theta_i = \theta^n)] = \frac{\frac{\phi(z-\Delta)}{\phi(z)}}{1 + \frac{\phi(z-\Delta)}{\phi(z)}}.$$

Therefore,

$$p = \frac{\frac{\phi(z-\Delta)}{\phi(z)}}{1 + \frac{\phi(z-\Delta)}{\phi(z)}}.$$

Solving for z we obtain

$$z = \frac{\ln\left[\frac{p}{1-p}\right]}{\Delta} + \frac{\Delta}{2} = \zeta\left(p, \Delta, 0\right)$$

proving that $\lim z_0^n(p) = \zeta\left(p, \Delta, 0\right)$ and therefore $\lim z_1^n(p) = \zeta\left(p, \Delta, 1\right)$.

Note that $\Pr\{P^n \leq p|v_i\} = \Pr\{Y^n(k+1) \leq p|v_i\}$. The central limit theorem implies that for each $\epsilon > 0$, there exists an N such that

$$\Phi\left(\zeta\left(p, \Delta, 0\right)\right) - \epsilon \leq \Pr\{Y^n(k+1) \leq p|v_0\} \leq \Phi\left(\zeta\left(p, \Delta, 0\right)\right) + \epsilon$$

for all $n > N$. As ϵ is arbitrary we find that $\lim \Pr\{Y^n(k+1) \leq p|v_0\} \to \Phi\left(\zeta\left(p, \Delta, 0\right)\right)$. Similarly, $\lim \Pr\{Y^n(k+1) \leq p|v_1\} \to \Phi\left(\zeta\left(p, \Delta, 1\right)\right)$.Therefore,

$$\lim \Pr\{Y^n(k+1) \leq p|V\} = \Phi\left(\zeta\left(p, \Delta, V\right)\right)$$

for each $p \in [v_0, v_1]$. □

Let $\lambda = \frac{p}{1-p}$ and notice that $\lambda = \Pr\left\{V = v_1|P = p\right\} / \Pr\left\{V = v_0|P = p\right\}$, that is, λ is the relative likelihoods of the two states conditional of observing a price equal to p. Rearranging we obtain $p = \frac{\lambda}{1+\lambda}$.Therefore,

$$\int_{v_0}^{v_1} p d\Phi\left(\zeta\left(p, \Delta, 0\right)\right) = \int_0^\infty \frac{\lambda}{1+\lambda} g(\lambda; -\frac{\Delta^2}{2}, \Delta) d\lambda$$

where $g(\lambda; \mu, \sigma)$ denotes the density function for a lognormal distribution $G(\lambda; \mu, \sigma)$ with location parameter μ and scale parameter σ.

Lemma 14. *If $CA \leq 1/4$, then the equation*

$$2CA = \int_0^\infty \frac{\lambda}{1+\lambda} g(\lambda; -\frac{\Delta^2}{2}, \Delta) d\lambda$$

has a unique solution $\Delta^*(CA)$ that is decreasing in CA. If $CA = 0$, then $\Delta^*(CA) = \infty$, and if $CA = 1/4$, then $\Delta^* = 0$. If $CA > 1/4$, then CA exceeds the righthand side of the equation for every $\Delta \geq 0$, i.e., the equation has no solution.

Proof. If $\hat{\Delta} > \Delta$, then $G(\lambda; -\frac{\Delta^2}{2}, \Delta)$ second order stochastic dominates $G(\lambda; -\hat{\Delta}^2/2, \hat{\Delta})$. This follows from the property that a lognormal distribution $G(\mu, \sigma)$ second order stochastically dominates another lognormal $G(\hat{\mu}, \hat{\sigma})$ if $(i)\mu > \hat{\mu}$; (ii) $\sigma < \hat{\sigma}$; and (iii) $\mu + \frac{\sigma^2}{2} \geq \hat{\mu} + \frac{\hat{\sigma}^2}{2}$ (see Levy (1973)).

The function $f(\lambda) = (\lambda v_1 + v_0) / (1 + \lambda)$ is an increasing, strictly concave function of λ. Therefore, second order stochastically dominance implies that if $\hat{\Delta} > \Delta$, then

$$\int_0^\infty \left(\frac{\lambda v_1 + v_0}{1+\lambda} - v_0 \right) g(\lambda; -\frac{\Delta^2}{2}, \Delta) d\lambda > \int_0^\infty \left(\frac{\lambda v_1 + v_0}{1+\lambda} - v_0 \right) g(\lambda; -\frac{\hat{\Delta}^2}{2}, \hat{\Delta}) d\lambda,$$

that is, the expression is increasing in Δ.

Note that if $\Delta = 0$, then $\mathbb{E}[P|v_0] = 1/2$ and if $\Delta \to \infty$, then $\mathbb{E}[P^n|v_0] \to 0$ (see the proof of Lemma 2). Therefore, the equation $2CA = \mathbb{E}[P|v_0]$ has a unique solution $\Delta^*(CA)$ and this solution is decreasing in CA if $CA \leq 1/4$ and the equation has no solution if $CA > 1/4$. □

Lemma 15. *If $CA < 1/4$, then $\Pr\{P \leq p|V\} = \Phi(\zeta(p, \Delta^*(CA), V))$ for all $p \in [0, 1]$. If $CA \geq 1/4$, then P^n converges is probability to $1/2$.*

Proof. We prove this lemma by looking at two distinct cases. Note that $\bar{F}(1/2|v_1; \mu^n) - \bar{F}(1/2|v_0; \mu^n) \geq 0$ for each n. Therefore, either we have
$\liminf \sqrt{n} \left(\bar{F}(1/2|v_1; \mu^n) - \bar{F}(1/2|v_0; \mu^n) \right) > 0$ or we have
$\lim \sqrt{n} \left(\bar{F}(1/2|v_1; \mu^n) - \bar{F}(1/2|v_0; \mu^n) \right) = 0$.

Case 1. Suppose $\liminf \sqrt{n} \left(\bar{F}(1/2|v_1; \mu^n) - \bar{F}(1/2|v_0; \mu^n) \right) > 0$. Pick any limit point $\Delta > 0$ of the sequence. Then Lemmata 11 and 12 together imply that $\lim \mathbb{E}[P^n|v_0] = 2CA$ along this sequence. Therefore,

$$\int_0^1 p d\Phi(\zeta(p, \Delta, 0)) = 2CA.$$

Lemma 14 this equation has a unique solution Δ^* proving that the sequence also has a unique limit point. Therefore, the unique price distribution is given by

$\Phi\left(\zeta\left(p, \Delta^*(CA), V\right)\right)$.

Case 2. If $\lim \sqrt{n}\left(\bar{F}(1/2|v_1;\mu^n) - \bar{F}(1/2|v_0;\mu^n)\right) = 0$, then Lemma 9 implies that $P^n \to 1/2$.

If $CB < 1/4$, then $\liminf \sqrt{n}\left(\bar{F}(1/2|v_1;\mu^n) - \bar{F}(1/2|v_0;\mu^n)\right) > 0$ by Lemma 9. Therefore, the result follows from Case 1.

If $CB \geq 1/4$, then we will argue below that $\lim \sqrt{n}\left(\bar{F}(1/2|v_1;\mu^n) - \bar{F}(1/2|v_0;\mu^n)\right) = 0$. Therefore, the result follows from Case 2.

Assume to the contrary that $\liminf \sqrt{n}\left(\bar{F}(1/2|v_1;\mu^n) - \bar{F}(1/2|v_0;\mu^n)\right) > 0$. Then, the limit price distribution is given by $\Phi\left(\zeta\left(p, \Delta^*(CB), V\right)\right)$, $\lim \sqrt{n}\left(\bar{F}(1/2|v_1;\mu^n) - \bar{F}(1/2|v_0;\mu^n)\right) = \Delta^*(CB) > 0$, and $\int_0^1 p d\Phi\left(\zeta\left(p, \Delta^*, 0\right)\right) = 2CA$ as shown in Case 1. However, Lemma 14 implies that the equation $\int_0^1 p d\Phi\left(\zeta\left(p, \Delta, 0\right)\right) = 2CA$ does not have a positive solution Δ if $CA \geq 1/4$ leading to a contradiction. □

Lemma 16. *For each Δ, let $G(\Delta)$ denote the price distribution . If $\Delta > \Delta'$, then $G(\Delta)$ is a mean preserving spread of $G\left(\Delta'\right)$. Therefore, Lemma 5 informativeness of the price is decreasing in CA.*

Proof. Note that $\int_0^1 x dG(x) = \int_0^1 x dG'(x)$. G is a mean preserving spread of G' if and only if $\int_0^x G(y)dy \geq \int_0^x G'(y)dy$ at each point x where $G(x) - G'(x) = 0$. Let x be a point where $G(x) - G'(x) = 0$. Integrating by parts implies that

$$\int_0^x G(y)dy = xG(x) - \int_0^x y g(y)dy$$
$$\int_0^x G(y)dy = xG(x) - \int_0^x \frac{g(y|v_1)}{g(y|v_1) + g(y|v_0)} \frac{g(y|v_1) + g(y|v_0)}{2} dy$$
$$\int_0^x G(y)dy = xG(x) - \frac{G(x|v_1)}{2}$$

Therefore, if $G(x) = G'(x)$, then

$$\int_0^x G(y)dy - \int_0^x G'(y)dy = \frac{G'(x|v_1) - G(x|v_1)}{2}.$$

Let $z(x)$ and $z'(x)$ denote the values according to P and P'. If $z(x) < z'(x)$, then $z(x) - \Delta < z'(x) - \Delta < z'(x) - \Delta'$. However,

$$G(x) = \frac{\Phi(z(x)) + \Phi(z(x) - \Delta)}{2} < G'(x) = \frac{\Phi(z'(x)) + \Phi(z'(x) - \Delta')}{2}$$

which is a contradiction. Hence, $z(x) \geq z'(x)$. This implies that $G(x|v_0) =$

$\Phi(z(x)) \geq G'(x|v_0) = \Phi(z'(x))$. However, $G(x) = G'(x)$ now implies that $G'(x|v_1) \geq G(x|v_1)$ which is the inequality that we were trying to show. The fact that $\Delta^*(CA)$ is decreasing in CA completes the argument. □

A.3. Proof of Theorem 3. We begin with some preliminary definitions. Given a symmetric equilibrium (μ, H),

$$u(\theta, v_i) = K \int_0^1 (\int_0^b (v_i - b)\,(1 - H(b|v_i))^{\lfloor \kappa n \rfloor - 1}\, H(b|v_i)^{n-1-\lfloor \kappa n \rfloor} dH(b|v_i))dH(b|\theta),$$

where $K = \binom{n-1}{1}\binom{n-2}{\lfloor \kappa n \rfloor - 1}$. In words, $H(\theta)$ is the equilibrium bidding strategy for type θ and $u(\theta, v_i)$ is the equilibrium payoff of type θ in state v_i from using the equilibrium strategy $H(\theta)$. If θ is a continuity point of the equilibrium type distribution F, then Lemma 1 implies that $H(b|\theta)$ puts probability one on the bid $b(\theta)$. Given that we are working with distributional strategies, $H(b|\theta)$ is not well defined for any $\theta \notin \text{supp } F$. For such types θ, define $H(b|\theta)$ such that $H(\{b = b(\theta)\}|\theta) = 1$.

Fix an equilibrium (μ, H) and let F denote the equilibrium type distribution. Convexity of C implies that

$$C(\mu) = C(F) = \int c(\theta')dF(\theta').$$

Pick any binary experiment $G_{\theta,\theta^*} \in \mathcal{F}$ that generates posterior θ with probability $q\,(\theta, \theta^*)$ and θ^* with probability $1 - q(\theta, \theta^*)$. Bayesian plausibility implies that

$$q(\theta, \theta^*) := \frac{1/2 - \theta^*}{\theta - \theta^*} \in [0, 1].$$

Consider the following optimization problem:

$$\max_{a \in [0,1]} U((1 - a)\,F + aG_{\theta,\theta^*}, H).$$

This maximization problem is solved at $a = 0$ because F is chosen optimally by each bidder in equilibrium. Optimality of F also implies that

$$\frac{d}{da} U\left((1 - a)F + aG_{\theta,\theta^*}, H\right)|_{a=0} \leq 0$$

for each $\theta \in [0, 1]$.

Claim 3. We have $\frac{d}{da} U\left((1 - a)F + aG_{\theta,\theta^*}, H\right)|_{a=0} = 0$ and therefore $U\left(G_{\theta,\theta^*}, H\right) = U\left(F, H\right)$ for each $\theta, \theta^* \in \text{supp } F$.

Proof. Pick an experiment $G_i \in \mathcal{F}$ such that $0 < F(\text{supp } G_i) < 1$ and

$$G_i(A) = \frac{\int_A dF(\theta)}{F(\text{supp } G_i)}$$

for every Borel set $A \subset [0,1]$. For any such G_i we have $U'((1-a)F + aG, H)|_{a=0} = 0$. This is because (1) if $U'((1-a)F + aG_i, H)|_{a=0} > 0$, then $U((1-a)F + aG_i, H) - U(F,H) > 0$ for all a sufficiently small, contradicting that F is optimal; (2) if $\frac{d}{da}U((1-a)F + aG_i, H)|_{a=0} < 0$, then $F + a(F - G_i) \in \mathcal{F}$ for all $a < F(\text{supp } G_i)$ and $U(F + a(F - G_i), H) - U(F,H) > 0$ for all sufficiently small a, again a contradiction.

Pick any sequence of such experiments G_i converging in the weak-star topology to the binary experiment G_{θ,θ^*} for $\theta, \theta^* \in \text{supp } F$. The fact that

$$\frac{d}{da}U((1-a)F + aG_i, H)|_{a=0} = 0$$

for each i along the sequence implies that

$$\frac{d}{da}U((1-a)F + aG_{\theta,\theta^*}, H)|_{a=0} = 0$$

for each $\theta, \theta^* \in \text{supp } F$. The equality $U(G_{\theta,\theta^*}, H) = U(F,H)$ follows
because $U((1-a)F + aG_{\theta,\theta^*}, H) = (1-a)U(F,H) + aU(G_{\theta,\theta^*}, H)$. □

Claim 3 implies that

$$q(\theta,\theta^*)(u(\theta) - c(\theta)) + (1 - q(\theta,\theta^*))(u(\theta^*) - c(\theta^*)) = U(F,H) \tag{A.3}$$

for $\theta, \theta^* \in \text{supp } F$.

Claim 4. The support of the equilibrium type distribution is an interval.

Proof. On the way to a contradiction assume that the type distribution is flat over an interval $[\theta_1, \theta_2]$ with $\theta_1, \theta_2 \in \text{supp } F$. Let $\bar{b}(\theta_1) = \max \text{supp} H(\theta_1)$, that is, $\bar{b}(\theta_1)$ is the highest bid in the support of type θ_1, and let $\underline{b}(\theta_2) = \min \text{supp} H(\theta_2)$. The fact that F is flat on $[\theta_1, \theta_2]$ and the fact that bidding is atomless and monotone implies that $u(b, v_i) = u(\bar{b}(\theta_1), v_i)$, for any $b \in [\bar{b}(\theta_1), \underline{b}(\theta_2)]$, in other words, any type $\theta \in [\theta_1, \theta_2]$ is indifferent between bid $\bar{b}(\theta_1)$ and any other bid $b \in [\bar{b}(\theta_1), \underline{b}(\theta_2)]$. We will prove the result by studying two mutually exclusive cases: 1) the type $1/2 \in (\theta_1, \theta_2)$, and 2) the type $1/2 \notin (\theta_1, \theta_2)$.

Suppose that $1/2 \notin (\theta_1, \theta_2)$. Without loss of generality further suppose that

$\theta_2 \leq 1/2$ and pick $\theta^* \in \text{supp } F$ with $\theta^* > 1/2$.[13] For any $\theta \in [\theta_1, \theta_2]$, consider the binary experiment G_{θ,θ^*}. Substituting for $q(\theta, \theta^*)$ in the first order condition with respect to G_{θ,θ^*} (Claim 3) and setting $u_i := u(\bar{b}(\theta_1), v_i)$ we obtain:

$$c(\theta) \geq u_0 + \theta (u_1 - u_0) + \frac{\theta - 1/2}{1/2 - \theta^*} (u(\theta^*) - c(\theta^*)) - \frac{\theta - \theta^*}{1/2 - \theta^*} U(F, H) \qquad \text{(A.4)}$$

for all $\theta \in [\theta_1, \theta_2]$ and with equality for $\theta = \theta_1, \theta_2$. The righthand-side of Inequality (A.4) is affine and can be expressed as $c(\theta) \geq \theta D + E$ for all $\theta \in [\theta_1, \theta_2]$ and $c(\theta_i) = \theta_i D + E$ for $i = 1, 2$, where D and E are constants independent of θ. Pick $\hat{\theta} = \lambda\theta_1 + (1 - \lambda)\theta_2$ where $\lambda \in (0, 1)$. Note that $\lambda c(\theta_1) + (1 - \lambda) c(\theta_2) = \hat{\theta} D + E$. Convexity of c implies that $c(\hat{\theta}) < \lambda c(\theta_1) + (1 - \lambda) c(\theta_2) = \hat{\theta} D + E$. However, this contradicts Inequality (A.4), which requires $c(\hat{\theta}) \geq \hat{\theta} D + E$.

Assume that $1/2 \in (\theta_1, \theta_2)$. Pick binary experiment G_{θ_1,θ_2}. Note that $\theta_1, \theta_2 \in \text{supp } F$. Therefore, Claim 3 implies that

$$q(\theta_1, \theta_2) (\theta_1 u_1 + (1 - \theta_1) u_0) + (1 - q(\theta_1, \theta_2)) (\theta_2 u_1 + (1 - \theta_2) u_0) \\ = q(\theta_1, \theta_2) c(\theta_1) + (1 - q(\theta_1, \theta_2)) c(\theta_2) + U(F, H).$$

Bayesian plausibility implies that $\theta_1 q(\theta_1, \theta_2) + (1 - q(\theta_1, \theta_2)) \theta_2 = 1/2$. Using this identity to rewrite the first order condition we obtain

$$\frac{u_1 + u_0}{2} - U(F, H) = q(\theta_1, \theta_2) c(\theta_1) + (1 - q(\theta_1, \theta_2)) c(\theta_2).$$

Similarly, the first order condition with respect to the uninformative experiment F° implies

$$\frac{u_1 + u_0}{2} - U(F, H) \leq c(1/2).$$

Therefore, $c(1/2) \geq q(\theta_1, \theta_2) c(\theta_1) + (1 - q(\theta_1, \theta_2)) c(\theta_2)$ contradicting the strict convexity of c. □

Given an equilibrium type distribution F, let $\underline{\theta} = \min \text{supp } F$ and $\bar{\theta} = \max \text{supp } F$. For each $\theta \in [\underline{\theta}, \bar{\theta}]$, not equal to $1/2$, define a binary experiment G_{θ,θ^*} as the experiment that generates posteriors θ and

$$\theta^* := \begin{cases} \underline{\theta} & \text{if } \theta > 1/2, \\ \bar{\theta} & \text{if } \theta < 1/2, \end{cases}$$

[13] This assumption is without loss because if instead $\theta_1 \geq 1/2$, we can choose $\theta^* \in \text{supp } F$ with $\theta^* < 1/2$ and proceed as in the argument.

with probabilities $q(\theta, \theta^*)$ and $1 - q(\theta, \theta^*)$. Note that $\theta, \theta^* \in \text{supp } F$ and therefore the first order condition with respect to G_{θ,θ^*} holds with equality for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

Claim 5. We have

$$u(\theta, v_1) - u(\theta, v_0) = c'(\theta) + D\left(\theta^*\right), \tag{A.5}$$

at every $\theta \in \text{supp } F$ where c is differentiable.

Suppose that θ is a continuity point of F. Note that $u(\theta) = \theta u(\theta, v_1) + (1-\theta)\, u(\theta, v_0)$. The fact that each type chooses its bid optimally (or, equivalently the envelope theorem) implies that

$$\theta \frac{d}{d\theta}\left(u(\theta, v_1)\right) + (1-\theta)\frac{d}{d\theta}\left(u(\theta, v_0)\right) = 0.$$

Therefore, $u'(\theta) = u(x, v_1) - u(x, v_0)$. To see this, define $u(\theta'|\theta) = \theta u(\theta'|v_1) + (1-\theta)\, u(\theta'|v_0)$ for any $\theta' \geq \theta$. Note that $u(\theta'|\theta)$ is maximized at $\theta' = \theta$ because type θ could always deviate to a bid submitted by type θ'. Therefore, $u'(\theta|\theta) = 0$.

Alternatively, θ is a discontinuity point of F. In this case, we will show that $u'_+(\theta) = \lim_{\theta' \downarrow \theta} u(\theta', v_1) - u(\theta', v_0)$ and $u'_-(\theta) = \lim_{\theta' \uparrow \theta} u(\theta', v_1) - u(\theta', v_0)$. Recall that $\bar{b}\left(\theta\right) = \max \text{supp} H(\theta)$. In this case, the envelope theorem again implies that

$$\theta \lim_{\theta' \downarrow \theta} \frac{u(\bar{b}\left(\theta\right), v_1) - u(\bar{b}\left(\theta'\right), v_1)}{\theta - \theta'} + (1-\theta) \lim_{\theta' \downarrow \theta} \frac{u(\bar{b}\left(\theta\right), v_0) - u(\bar{b}\left(\theta'\right), v_0)}{\theta - \theta'} = 0.$$

Therefore, $u'_+(\theta) = \lim_{\theta' \downarrow \theta} u(\theta', v_1) - u(\theta', v_0)$. The argument for $u'_-(\theta)$ is analogous.

For each $\theta, \theta' \in (1/2, \bar{\theta})$ we have $U\left(G_{\theta,\theta^*}, H\right) = U\left(G_{\theta',\theta^*}, H\right) = U\left(F, H\right)$ by Claim 3. Therefore, $\frac{1/2-\theta^*}{\theta-\theta^*}$

$$u(\theta) - c(\theta) = \frac{U\left(F, H\right)}{q(\theta, \theta^*)} + \left(c(\theta^*) - u(\theta^*)\right)\frac{\left(1 - q(\theta, \theta^*)\right)}{q(\theta, \theta^*)}$$
$$u(\theta') - c(\theta') = \frac{U\left(F, H\right)}{q(\theta', \theta^*)} + \left(c(\theta^*) - u(\theta^*)\right)\frac{\left(1 - q(\theta', \theta^*)\right)}{q(\theta', \theta^*)}$$

Taking the difference of the two equations and substituting in for $q(\theta, \theta^*)$ we obtain

$$\frac{u(\theta) - c(\theta) - \left(u(\theta') - c(\theta')\right)}{\theta - \theta'} = \frac{U(F, H) + \left(c(\theta^*) - u(\theta^*)\right)}{1/2 - \theta^*}.$$

Since this is true for each θ and $\theta' \in (1/2, \bar{\theta})$, we find that

$$\frac{d}{d\theta}\left(u(\theta) - c(\theta)\right) = \frac{U(F, H) + \left(c(\theta^*) - u(\theta^*)\right)}{1/2 - \theta^*}$$

The fact that c is convex implies that the left and right derivative of this function exists. Therefore,

$$u'_+(\theta) = c'_+(\theta) + \frac{U(F,H) + (c(\theta^*) - u(\theta^*))}{1/2 - \theta^*},$$
$$u'_-(\theta) = c'_-(\theta) + \frac{U(F,H) + (c(\theta^*) - u(\theta^*))}{1/2 - \theta^*}.$$

Rewriting, we obtain $u(\theta, v_1) - u(\theta, v_0) = c'(\theta) + D(\theta^*)$ at any θ where c is differentiable.

Claim 6. The type distribution does not have an atom at $\theta \in (\underline{\theta}, \bar{\theta})$ if and only if $c(\theta)$ is differentiable at θ.

Proof. The function $u(\theta, v_1) - u(\theta, v_0)$ is discontinuous at $\theta \in (\underline{\theta}, \bar{\theta})$, or equivalently, $u'_+(\theta) \neq u'_-(\theta)$ if and only if the type distribution has an atom at θ by Lemma 1. The claim now follows because Equation (A.5) shows that $u(\theta, v_1) - u(\theta, v_0)$ is equal to $c'(\theta)$ plus a constant. □

The convex function c is twice continuously differentiable almost everywhere by Alexandrov's Theorem. Therefore, the type distribution is continuous at almost every $\theta \in (\underline{\theta}, \bar{\theta})$ and

$$u(\theta, v_i) = \int_0^\theta (v_i - b(\theta'))\, G(\theta'|v_i) d\theta'.$$

Taking the derivative of Equation (A.5) with respect to θ we find

$$(1 - b(\theta))\, G(\theta|v_1) + b(\theta) G(\theta|v_0) = c''(\theta)$$

for almost every θ, proving the theorem.

A.4. Proof of Theorem 4. Pick a ball $N(F^\circ) \subset \mathcal{F}$ where non-redundancy and ρ-monotonicity hold. Let F denote any binary experiment in $N(F^\circ)$ that generates posteriors in $\{1-q, q\}$ with equal probability and let $\lambda := q/(1-q) < 1$ denote this experiment's likelihood ratio. Consider another binary experiment $F_1 \in \mathcal{F}$ with likelihood ratio $\sqrt{\lambda}$. Note that $F_1 \in N(F^\circ)$.

The experiment $F_1 \otimes F_1$ generates posterior likelihood ratios $(1/\lambda, 1, \lambda)$ with probabilities $(x/2, 1-x, x/2)$, where $x(\lambda) = (1+\lambda) / \left(1 + \sqrt{\lambda}\right)^2$. This distribution over posteriors is equivalent to the experiment $(1 - x(\lambda))F^\circ + x(\lambda)F$ and therefore $F_1 \otimes F_1 \in N(F^\circ)$. Non-redundancy implies that $C(F_1 \otimes F_1) \leq x(\lambda)C(F)$ and ρ-monotonicity implies that $C(F_1 \otimes F_1) \geq \rho C(F_1)$. Therefore, $c(F_1) \leq x(\lambda)c(F)/\rho$.

Similarly, if we take F_2 to denote the binomial experiment with likelihood ratio $\lambda^{1/4}$ we find that

$$C(F_2) \leq x(\sqrt{\lambda})c(F_1)/\rho \leq x(\lambda)C(F_1)/\rho,$$

where the last inequality follows from the fact that $x(\lambda)$ is increasing in λ. Therefore, $C(F_2) \leq (x(\lambda)/\rho)^2 C(F)$ and proceeding recursively we obtain $C(F_j) \leq (x(\lambda)/\rho)^j C(F)$ for any binary experiment $F \in N(F^\circ)$ and binary experiment F_j with likelihood ratio $\lambda^{(1/2)^j}$.

We now choose a particular binary experiment $\hat{F} \in N(F^\circ)$ with likelihood ratio λ sufficiently close to one such that $2x(\lambda)/\rho < 1$. We can indeed pick λ in this way because $\rho > 1$ by assumption, $x(1) = 1/2$, and $x(\lambda)$ is continuous. Note that $\bar{F}(1/2|v_1) - \bar{F}(1/2|v_0) = \lambda - 1$. Therefore,

$$CA_{\hat{F}_j} \leq C(\hat{F}) \frac{(x(\lambda)/\rho)^j}{\lambda^{(1/2)^j} - 1}.$$

Taking the limit as $n \to \infty$, we find

$$CA \leq C(\hat{F}) \lim_j \frac{(x(\lambda)/\rho)^j}{\lambda^{(1/2)^j} - 1} = C(\hat{F}) \lim_j \frac{(x(\lambda)/\rho)^j \ln(x(\lambda)/\rho)}{\lambda^{(1/2)^j} \ln(1/2) \ln \lambda \, (1/2)^j} = \lim_j \left(\frac{2x(\lambda)}{\rho}\right)^j = 0.$$

However, $CA = 0$ implies that information is aggregated.

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